PERMUTATIONS AND COMBINATIONS

1. The total number of selections of fruit which can be made from 3 bananas, 4 apples and 2 oranges, is
   a) 39　b) 315　c) 512　d) None of these
2. If $m = {}^{n}C_2$, then ${}^{m}C_2$ is equal to
   a) $3\,{}^{n}C_4$　b) ${}^{n+1}C_4$　c) $3\,{}^{n+1}C_4$　d) $3.\,{}^{n+1}C_3$
3. Five balls of different colours are to be placed in three boxes of different sizes. Each box can hold all five balls. In how many ways can we place the balls so that no box remains empty?
   a) 50　b) 100　c) 150　d) 200
4. The interior angles of a regular polygon measure 160° each. The number of diagonals of the polygon are
   a) 97　b) 105　c) 135　d) 146
5. There are n number of sets and m number of people have to be seated, then how many ways are possible to do this $(m < n)$?
   a) ${}^{n}P_m$　b) ${}^{n}C_m$　c) ${}^{n}C_n \times (m-1)!$　d) ${}^{n-1}P_{m-1}$
6. $\sum_{r=0}^{m} {}^{n+r}C_n$ is equal to
   a) ${}^{n+m+1}C_{n+1}$　b) ${}^{n+m+2}C_n$　c) ${}^{n+m+3}C_{n-1}$　d) None of these
7. The set $S = \{1, 2, 3, \dots, 12\}$ is to be partitioned into three sets A, B, C of equal size
   Thus, $A \cup B \cup C = S$
   $A \cup B = B \cup C = A \cup C = \emptyset$
   The number of ways to partition S is
   a) $12!/3!\,(4!)^3$　b) $12!/3!\,(3!)^4$　c) $12!/(4!)^3$　d) $12!\,(3!)^4$
8. The number of selecting at least 4 candidates from 8 candidates is
   a) 270　b) 70　c) 163　d) None of these
9. The letters of the word COCHIN are permuted and all the permutations are arranged in an alphabetical order as in an English dictionary. The number of words that appear before the word COCHIN is
   a) 360　b) 192　c) 96　d) 48
10. There are 5 roads leading to0 a town from a village. The number of different ways in which a village can go to the town and return back, is
    a) 20　b) 25　c) 5　d) 10
11. An n-digit number is a positive number with exactly n digits. Nine hundred distinct n-digit numbers are to be formed using only the three digits 2, 5 and 7. The smallest value of n for which this is possible, is
    a) 6　b) 7　c) 8　d) 9
12. If ${}^{n}C_{r-1} = 36$, ${}^{n}C_r = 84$ and ${}^{n}C_{r+1} = 126$, then
    a) $n = 8, r = 4$　b) $n = 9, r = 3$　c) $n = 7, r = 5$　d) None of these
13. The total number of ways in which 11 identical apples can be distributed among 6 children is
    a) 252　b) 462　c) 42　d) None of these
14. A polygon has 44 diagonals, then the number of its sides are
    a) 11　b) 7　c) 8　d) None of these
15. The number of ways in which 12 balls can be divided between two friends, one receiving 8 an the other 4, is
    a) $\frac{12\,!}{8\,!\,4\,!}$　b) $\frac{12\,!\,2\,!}{8\,!\,4\,!}$　c) $\frac{12\,!}{8\,!\,4\,!\,2\,!}$　d) None of these

16. Assuming that no two consecutive digits are same. The number of n digit numbers is
a) $n!$ b) $9!$ c) 9^n d) n^9
17. The figure 4, 5, 6, 7, 8 are written in every possible order. The number of numbers greater than 56000 is
a) 72 b) 96 c) 90 d) 98
18. p points are chosen on each of the three coplanar lines. The maximum number of triangles formed with vertices at these points is
a) p^3+3p^2 b) $\frac{1}{2}(p^3+p)$ c) $\frac{p^2}{2}(5p-3)$ d) $p^2(4p-3)$
19. Number of divisors of the form $(4n+2), n \geq 0$ of the integer 240 is
a) 4 b) 8 c) 10 d) 3
20. How many different words can be formed by jumbling the letters in the word MISSISSIPPI in which no two S are adjacent?
a) $7.\,{}^6C_4.\,{}^8C_4$ b) $8.\,{}^6C_4.\,{}^7C_4$ c) $6.7.\,{}^8C_4$ d) $6.8.\,{}^7C_4$
21. Let l_1 and l_2 be two lines intersecting at P. If A_1, B_1, C_1 are points on l_1 and A_2, B_2, C_2, D_2, E_2 are points on l_2 and if none of these coincides with P, then the number of triangles formed by these eight points, is
a) 56 b) 55 c) 46 d) 45
22. There are two urns. Urn A has 3 distinct red balls and urn B has 9 distinct blue balls. From each urn two balls are taken out at random and then transferred to the other. Then number of ways in which this can be done, is
a) 3 b) 36 c) 66 d) 108
23. The number of words that can be formed out of the letters of the words 'ARTICE' so that the vowels occupy even places, is
a) 574 b) 36 c) 754 d) 144
24. The value of $2^n[1.3.5 \dots (2n-3)(2n-1)]$ is
a) $\frac{(2n)!}{n!}$ b) $\frac{(2n)!}{2^n}$ c) $\frac{n!}{(2n)!}$ d) None of these
25. The number of ways in which one can post 5 letters in 7 letters boxes is
a) 35 b) ${}^7P_5$ c) 7^5 d) 5^7
26. A car will hold 2 in the front seat and 1 in the rear seat. If among 6 persons 2 can drive, then number of ways in which the car can be filled, is
a) 10 b) 20 c) 30 d) None of these
27. The number of ways of selecting 10 balls from unlimited number of red, black, white and green balls is
a) 286 b) 84 c) 715 d) None of these
28. In a college examination, a candidates is required to answer 6 out of 10 questions which are divided into two sections each containing 5 questions. Further the candidate is not permitted to attempt more than 4 questions from either of the section. The number of ways in which he can make up a choice of 6 questions, is
a) 200 b) 150 c) 100 d) 50
29. In a cricket championship there are 36 matches. The number of terms, if each plays 1 match with other are
a) 9 b) 10 c) 8 d) 12
30. The number of all four digit numbers which are divisible by 4 that can be formed from the digits 1,2,3,4 and 5 is
a) 125 b) 30 c) 95 d) None of these
31. The number of committees of 5 persons consisting of at least one female member, that can be formed from 6 males and 4 females, is
a) 246 b) 252 c) 6 d) None of these
32. The number of ways that 8 beads of different colures be string as a necklace, is
a) 2520 b) 2880 c) 5040 d) 4320
33. 9 balls are to be placed in 9 boxes and 5 of the balls cannot fit into 3 small boxes. The number of ways of arranging one ball in each of the boxes is

a) 18720 b) 18270 c) 17280 d) 12780

34. The expression ${}^{n}C_r + 4 \cdot {}^{n}C_{r-1} + 6 \cdot {}^{n}C_{r-2} + 4 \cdot {}^{n}C_{r-3} + {}^{n}C_{r-4}$ equals
a) ${}^{n+4}C_r$ b) $2 \cdot {}^{n+4}C_{r-1}$ c) $4 \cdot {}^{n}C_r$ d) $11 \cdot {}^{n}C_r$

35. The total number of natural numbers of six digits that can be made with digits 1, 2, 3, 4, if all digits are to appear in the same number at least once, is
a) 1560 b) 840 c) 1080 d) 480

36. m men and n women are to be seated in a row so that no two women sit together. If $m > n$, then the number of ways in which they can be seated, is
a) $\frac{m!\,(m+1)!}{(m-n+1)!}$ b) $\frac{m!\,(m-1)!}{(m-n+1)!}$ c) $\frac{(m-1)!\,(m+1)!}{(m-n+1)!}$ d) None of these

37. If $a, b, c \in N$,The anumber of points having position vector $a\hat{\mathbf{i}} + b\hat{\mathbf{j}} + c\hat{\mathbf{k}}$ such that $6 \le a + b + c \le 10$, is
a) 110 b) 116 c) 120 d) 127

38. If ${}^{n+2}C_8 : {}^{n-2}P_4 = 57 : 16$, then the value of n is
a) 20 b) 19 c) 18 d) 17

39. There are 10 points in a plane, out of these 6 are collinear. The number of triangles formed by joining these points is
a) 100 b) 120 c) 150 d) None of these

40. A dictionary is printed consisting of 7 letters words only that can be made with a letters of the word CRICKET. If the words are printed at the alphabetical order as in an ordinary dictionary, then the number of words before the word CRICKET is
a) 530 b) 480 c) 531 d) 481

41. A man has 7 friends. In how many ways he can invite one or more of them for a tea party?
a) 128 b) 256 c) 127 d) 130

42. In how many ways can Rs 16 be divided into 4 persons when none of them get less than Rs 3?
a) 70 b) 35 c) 64 d) 192

43. The number of divisors of the number of 38808 (excluding 1 and the number itself)is
a) 70 b) 72 c) 71 d) None of these

44. The number of 4-digit even numbers that can be formed using 0,1,2,3,4,5,6 without repetition is
a) 120 b) 300 c) 420 d) 20

45. If ${}^{n}P_r = 30240$ and ${}^{n}C_r = 252$, then the ordered pair (n, r) is equal to
a) (12, 6) b) (10, 5) c) (9, 4) d) (16, 7)

46. ${}^{n}C_r + 2\,{}^{n}C_{r-1} + {}^{n}C_{r-2}$ is equal to
a) ${}^{n+1}C_r$ b) ${}^{n+1}C_{r+1}$ c) ${}^{n+2}C_r$ d) ${}^{n+2}C_{r+1}$

47. How many nine digit numbers can be formed by using the digits 2,2,3,3,5,5,8,8,8 so that the odd digits occupy even positions?
a) 7560 b) 180 c) 16 d) 60

48. The letters of the word RANDOM are written in all possible orders and these words are written out as in a dictionary, then the rank of the word RANDOM is
a) 614 b) 615 c) 613 d) 616

49. From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on the shelf so that the dictionary is always in the middle. Then the number of such arrangement is
a) At least 500 but less than 750 b) At least 750 but less than 1000
c) At least 1000 d) Less than 500

50. Three dice are rolled. The number of possible outcomes in which at least one die shows 5 is
a) 215 b) 36 c) 125 d) 91

51. The number of ways in which thirty five apples can be distributed among 3 boys so that each can have any number of apples, is
a) 1332 b) 666 c) 333 d) None of these

52. The smallest value of x satisfying the inequlity ${}^{10}C_{x-1} > 2 \cdot {}^{10}C_x$ is

a) 7 b) 10 c) 9 d) 8

53. The number of ways in which 5 pictures can be hung from 7 picture nails on the wall is
a) 7^5 b) 5^7 c) 2520 d) None of these

54. If eight persons are to address a meeting, then the number of ways in which a specified speaker is to speak before another specified speaker is
a) 2520 b) 20160 c) 40320 d) None of these

55. The number of ways in which five identical balls can be distributed among ten identical boxes such that no box contains more than one ball, is
a) 10! b) $\frac{10!}{5!}$ c) $\frac{10!}{(5!)^2}$ d) None of these

56. The number of numbers of four different digits that can be formed from the digits of the number 12356 such that the numbers are divisible by 4, is
a) 36 b) 48 c) 12 d) 24

57. A parallelogram is cut by two sets of m lines parallel to its sides. The number of parallelograms thus formed is
a) $({}^{m}C_2)^2$ b) $({}^{m+1}C_2)^2$ c) $({}^{m+2}C_2)^2$ d) None of these

58. Sum of all the odd divisors of 720 is
a) 76 b) 78 c) 80 d) 84

59. How many words can be formed from the letters of the word DOGMATIC, if all the vowels remain together?
a) 4140 b) 4320 c) 432 d) 43

60. The number of times the digit 5 will be written when listing the integers from 1 to 1000, is
a) 271 b) 272 c) 300 d) None of these

61. The number of ways of painting the faces of a cube with six different colours is
a) 1 b) 6 c) 6 ! d) None of these

62. How many four digit numbers can be formed using the digits 1, 2, 3, 4, 5 such that at least one of the digit is repeated?
a) $4^4 - 5!$ b) $4^5 - 4!$ c) $5^4 - 4!$ d) $5^4 - 5!$

63. The number of ways of arranging 8 men and 4 women around a circular table such that no two women can sit together, is
a) 8! b) 4! c) 8! 4! d) $7!\ {}^{8}P_4$

64. Let A be the set of 4 digit number $a_1a_2a_3a_4$, where $a_1 < a_2 < a_3 < a_4$, then $n(A)$ is equal to
a) 84 b) 126 c) 210 d) None of these

65. If ${}^{56}P_{r+6} : {}^{54}P_{r+3} = 30800 : 1$, then the value of r is
a) 40 b) 51 c) 41 d) 510

66. The number of numbers of 9 distinct digits such that all the digits in the first four places are less than the digit in the middle and all the digits in the last four places are greater than that in the middle is
a) 48 b) 576 c) 8! d) None of these

67. We are to form different words with the letters of the word INTEGER. Let m_1 be the number of words in which I and N are never together and m_2 be the number of words which begin with I and end with R, then m_1/m_2 is equal to
a) 30 b) 60 c) 90 d) 180

68. A polygon has 170 diagonals. How many sides will it have?
a) 12 b) 17 c) 20 d) 25

69. Out of 10 consonants four vowels, the number of words that can be formed using six consonants and three vowels
a) ${}^{10}P_6 \times {}^{6}P_3$ b) ${}^{10}C_6 \times {}^{6}C_3$ c) ${}^{10}C_6 \times {}^{4}C_3 \times 9!$ d) ${}^{10}P_6 \times {}^{4}P_3$

70. The total number of all proper factors of 75600 is
a) 120 b) 119 c) 118 d) None of these

71. There are 5 letters and 5 different envelopes. The number of ways in which all the letters can be put in wrong envelope, is

a) 119 b) 44 c) 59 d) 40

72. If ${}^{35}C_{n+7} = {}^{35}C_{4n-2}$, then all the values of n are given by

a) 28 b) 3, 6 c) 3 d) 6

73. There are 3 candidates for a post and one is to be selected by the votes of 7 men. The number of ways in which votes can be given, is

a) 7^3 b) 3^7 c) ${}^7C_3$ d) None of these

74. The number of all the possible selections which a student can make for answering one or more questions out of eight given questions in a paper, when each question has an alternative is

a) 256 b) 6560 c) 6561 d) None of these

75. In a conference of 8 persons, if each person shake hand with the other one only, then the total number of shake hands shall be

a) 64 b) 56 c) 49 d) 28

76. A person is permitted to select at least one and at most n coins from a collection of $2n + 1$ (distinct) coins. If the total number of ways in which he can select coins is 255, then n equals

a) 4 b) 8 c) 16 d) 32

77. How many words can be made from the letters of the word 'COMMITTEE'?

a) $\frac{9!}{(2!)^2!}$ b) $\frac{9!}{(2!)^3!}$ c) $\frac{9!}{2!}$ d) 9!

78. Ramesh has 6 friends. In how many ways can he invite one or more of them at a dinner?

a) 61 b) 62 c) 63 d) 64

79. How many different non-digit numbers can be formed from the digits of the number 223355888 by rearrangement of the digits so that the odd digits occupy even places?

a) 16 b) 36 c) 60 d) 180

80. In how many ways a garland can be made from exactly 10 flowers?

a) 10! b) 9! c) 2(9!) d) $\frac{9!}{2}$

81. The number of ways in which 20 different pearls of two colours can be set alternately on a necklace, there being 10 pearls of each colour, is

a) $9! \times 10!$ b) $5 \times (9!)^2$ c) $(9!)^2$ d) None of these

82. Seven women and seven men are to sit round a circular table such that there is a man on either side of every women; the number of seating arrangements is

a) $(7\,!)^2$ b) $(6\,!)^2$ c) $6\,! \times 7\,!$ d) $7\,!$

83. The number of ways in which 9 persons can be divided into three equal groups is

a) 1680 b) 840 c) 560 d) 280

84. The number of ways in which four letters can be selected from the word *degree* is

a) 7 b) 6 c) $\frac{6\,!}{3\,!}$ d) None of these

85. If the letters of the word 'SACHIN' are arranged in all possible ways and these words are written out as in dictionary, then the word 'SACHIN' appears at serial number

a) 602 b) 603 c) 600 d) 601

86. If ${}^{k+5}P_{k+1} = \frac{11(k-1)}{2} \cdot {}^{k+3}P_k$, then the values of k are

a) 7 and 11 b) 6 and 7 c) 2 and 11 d) 2 and 6

87. In an examination there are three multiple choice questions and each question has 4 choices. Number of ways in which a student can fail to get all answers correct, is

a) 11 b) 12 c) 27 d) 63

88. A lady given a dinner party for six guest. The number of ways in which they may be selected from among ten friends, if two of the friends will not, attends the party together is

a) 112 b) 140 c) 164 d) None of these

89. Which of the following is incorrect?
a) ${}^nC_r = {}^nC_{n-r}$
b) ${}^nC_r = {}^{n-1}C_r + {}^nC_{n-r}$
c) ${}^nC_r = {}^{n-1}C_r + {}^{n-1}C_{r-1}$
d) $r!\ {}^nC_r = {}^nP_r$

90. The number of triangles that can be formed by 5 points in a line and 3 points on a parallel line is
a) ${}^8C_3$ b) ${}^8C_3 - {}^5C_3$ c) ${}^8C_3 - {}^5C_3 - 1$ d) None of these

91. Eight chair are numbered 1 to 8. Two women and three men wish to occupy one chair each. First the women choose the chairs from amongst the chair marked 1 to 4; and then the men select the chairs from amongst the remaining. The number of possible arrangement is
a) ${}^6C_3 \times {}^4C_2$ b) ${}^4P_2 \times {}^6P_3$ c) ${}^4C_2 + {}^4P_3$ d) None of these

92. Out of 8 given points, 3 are collinear. How many different straight lines can be drawn by joining any two points from those 8 points?
a) 26 b) 28 c) 27 d) 25

93. A five digit number divisible by 3 is to be formed using the numbers 0, 1, 2, 3, 4 and 5, without repetition. The total number of ways this can be done, is
a) 216 b) 240 c) 600 d) 3125

94. The number of five digits numbers that can be formed without any restriction, is
a) 990000 b) 100000 c) 90000 d) None of these

95. How many words can be formed from the letters of the word ARTICLE, if vowels always comes at the odd places?
a) 60 b) 576 c) $\frac{7!}{3!}$ d) 120

96. The number of divisors of 9600 including 1 and 9600 are
a) 60 b) 58 c) 48 d) 46

97. Let 1, 2, 3, 4 are four numbers. How many numbers can be made using all four numbers?
a) 1 b) 3 c) 2 d) 64

98. The total number of flags with three horizontal strips, in order that can be formed using 2 identical red, 2 identical green and 2 identical white strips, is equal to
a) 4! b) 3(4!) c) 2(4!) d) None of these

99. The total number of proper divisors of 38808 is
a) 72 b) 70 c) 69 d) 71

100. How many 10 digits numbers can be written by using digits 9 and 2?
a) ${}^{10}C_1 \times {}^9C_2$ b) 2^{10} c) ${}^{10}C_2$ d) 10!

101. Let $A = \{x_1, x_2, x_3, x_4, x_5, x_6\}$,
$B = \{y_1, y_2, y_3, y_4, y_5, y_6\}$.Then the number of one –one mapping from A to B such that $f(x_i) \neq y_v i = 1, 2, 3, 4, 5, 6$ is
a) 720 b) 265 c) 360 d) 145

102. A man invites a party to $(m+n)$ friends to dinner and places m at one round table and n at another. The number of ways of arranging the guests is
a) $\frac{(m+n)!}{m!\,n!}$ b) $\frac{(m+n)!}{(m-1)!\,(n-1)!}$ c) $(m-1)!\,(n-1)!$ d) None of these

103. The number of ways in which seven persons can be arranged at a round table, if two particular persons may not sit together is
a) 480 b) 120 c) 80 d) None of these

104. If ${}^{2n+1}P_{n-1} : {}^{2n-1}P_n : 3 : 5$, then the value of n is equal to
a) 4 b) 3 c) 2 d) 1

105. The number of ways in which a committee can be formed of 5 members from 6 men and 4 women if the committee has at least one woman, is
a) 186 b) 246 c) 252 d) 244

106. In how many ways can 5 books be selected out of 10 books, if two specific books are never selected?
a) 56 b) 65 c) 58 d) None of these

107. The number of parallelograms that can be formed from a set of four parallel lines intersecting another set of three parallel lines, is
a) 6 b) 18 c) 12 d) 9

108. There is a set of m parallel lines intersecting a set of another n parallel lines in a plane. The number of parallelograms formed, is
a) ${}^{m-1}C_2 . {}^{n-1}C_2$ b) ${}^{m}C_2 . {}^{n}C_2$ c) ${}^{m-1}C_2 . {}^{n}C_2$ d) ${}^{m}C_2 . {}^{n-1}C_2$

109. The value of ${}^{50}C_4 + \sum_{r=1}^{6} {}^{56-r}C_3$ is
a) ${}^{56}C_4$ b) ${}^{56}C_3$ c) ${}^{55}C_3$ d) ${}^{55}C_4$

110. The number of numbers of 4 digits which are not divisible by 5, are
a) 7200 b) 3600 c) 14400 d) 1800

111. 4 buses runs between Bhopal and Gwalior. If a man goes from Gwalior to Bhopal by a bus and comes back to Gwalior by another bus, then the total possible ways are
a) 12 b) 16 c) 4 d) 8

112. The total number of different combinations of letters which can be made from the letters of the word MISSISSIPPI is
a) 150 b) 148 c) 149 d) None of these

113. Six points in a plane be joined in all possible ways by indefinite straight lines and if no two of them be coincident or parallel, and no three pass through the same point (with the exception of the original 6 points). The number of distinct points or intersection is equal to
a) 105 b) 45 c) 51 d) None of these

114. The total numbers of ways of dividing 15 things into groups of 8,4 and 3 respectively is
a) $\frac{15!}{8!4!(3!)^2}$ b) $\frac{15!}{8!4!3!}$ c) $\frac{15!}{8!4!}$ d) None of these

115. In a circus there are ten cages for accommodating ten animals. Out of these four cages are so small that five out of 10 animals cannot enter into them. In how many ways will it be possible to accommodate ten animals in these ten cages?
a) 66400 b) 86400 c) 96400 d) None of these

116. Let T_n denote the number of triangles which can be formed using the vertices of a regular polygon of n sides. If $T_{n+1} - T_n = 21$, then n equals
a) 5 b) 7 c) 6 d) 4

117. At an electron, a voter may vote for any number of candidates not greater than the number to be elected. There are 10 candidates and 4 are to be elected. If a voter votes for at least one candidate, then the number of ways in which he can vote, is
a) 6210 b) 385 c) 1110 d) 5040

118. All possible two factors products are formed from numbers 1, 2, 3, 4,...,200. The number of factors out of the total obtained which are multiples of 5, is
a) 5040 b) 7180 c) 8150 d) None of these

119. If the total number of m elements subsets of the set $A = \{a_1, a_2, a_3, ... a_n\}$ is λ times the number of 3 elements subsets containing a_4,then n is
a) $(m-1)\lambda$ b) $m\lambda$ c) $(m+1)\lambda$ d) 0

120. The number of natural numbers less than 1000, in which no two digits are replaced, is
a) 738 b) 792 c) 837 d) 720

121. If ${}^{n}C_r$ denotes the number of combinations of n things takes r at a time, then the expression ${}^{n}C_{r+1} + {}^{n}C_{r-1} + 2 \times {}^{n}C_r$, equals
a) ${}^{n+2}C_r$ b) ${}^{n+2}C_{r+1}$ c) ${}^{n+1}C_r$ d) ${}^{n+1}C_{r+1}$

122. If $\frac{2}{9!} + \frac{2}{3!7!} + \frac{1}{5!5!} = \frac{2^a}{b!}$,where $a, b, \in N$, then the ordered pair (a, b) is
a) (9, 10) b) (10, 9) c) (7, 10) d) (10, 7)

123. The number of diagonals that can be drawn by joining the vertices of an octagon is

a) 28 b) 48 c) 20 d) None of these

124. A father with 8 children takes 3 at a time to the zoological garden, as often as he can without taking the same 3 children together more than once. The number of times he will go to the garden, is

a) 112 b) 56 c) 336 d) None of these

125. If ${}^{189}C_{35} + {}^{189}C_x = {}^{190}C_x$, then x is equal to

a) 34 b) 35 c) 36 d) 37

126. The number of ways in which n ties can be selected from a rack displaying 3 n different ties is

a) $\frac{3n\,!}{2n\,!}$ b) $3 \times n\,!$ c) $(3n)\,!$ d) $\frac{3n\,!}{n\,!\,2n\,!}$

127. The number of permutations of 4 letters that can be made out of the letters of the word EXAMINATION is

a) 2454 b) 2452 c) 2450 d) 1806

128. The number of ways in which 5 boys and 5 girls can be seated for a photograph so that no two girls sit next to each other is

a) $6!.5!$ b) $(5!)^2$ c) $\frac{10!}{(5!)}$ d) $\frac{10!}{(5!)^2}$

129. The number of diagonals of a polygon of 20 sides is

a) 210 b) 190 c) 180 d) 170

130. The value of ${}^{47}C_4 + \sum_{r=1}^{5} {}^{52-r}C_3$is equal to

a) ${}^{47}C_6$ b) ${}^{52}C_5$ c) ${}^{53}C_4$ d) None of these

131. In how many ways can 21 English and 19 Hindi books be placed in a row so that no two Hindi books are together?

a) 1540 b) 1450 c) 1504 d) 1405

132. In a group of boys, two boys are brothers and in this group, 6 more boys are there. In how many ways, they can sit if the brothers are not to sit alongwith each other :

a) 4820 b) 1410 c) 2830 d) None of these

133. All possible four-digit numbers are formed using the digits 0,1,2,3 so that no number has repeated digits. The number of even number among them is

a) 9 b) 18 c) 10 d) None of these

134. In how many ways can 4 prizes be distributed among 3 students, if each students can get all the 4 prizes?

a) 4! b) 3^4 c) $3^4 - 1$ d) 3^3

135. In a chess tournament where the participants were to play one game with one another, two players fell ill having played 6 games each, without playing among themselves. If the total number of games is 117, then the number of participants at the beginning was

a) 15 b) 16 c) 17 d) 18

136. How many even numbers of 3 different digits can be formed from the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 (repetition of digits is not allowed)?

a) 224 b) 280 c) 324 d) None of these

137. If a denotes the number of permutations of $x + 2$ things taken all at a time, b the number of permutations of x things taken 11 at a time and c the number of permutations of $x - 11$ things taken all at a time such that $a = 182\ bc$, then the value of x is

a) 15 b) 12 c) 10 d) 18

138. Eleven books consisting of 5 Mathematics, 4 physics and 2 Chemistry are places on a shelf. The number of possible ways of arranging them on the assumption that the books of the same subject are all together, is

a) 4! 2! b) 11! c) 5! 4! 3! 2! d) None of these

139. The number of mappings (functions) from the set $A = \{1, 2, 3\}$ into the set $B = \{1, 2, 3, 4, 5, 6, 7\}$ such that $f(i) \leq f(j)$ whenever $i < j$, is

a) 84 b) 90 c) 88 d) None of these

140. The number of ordered triplets of positives integers which are solutions of the equations of the equation $z + y + z = 100$, is

a) 6005 b) 4851 c) 5081 d) None of these

141. The number of four-letter words that can be formed (the words need not be meaningful) using the letters of the word MEDITERRANEAN such that the first letter is E and the last letter is R, is

a) $\frac{11!}{2!\,2!\,2!}$ b) 59 c) 56 d) $\frac{11!}{3!\,2!\,2!}$

142. A person goes for an examination in which there are four papers with a maximum of m marks from each paper. The number of ways in which one can get $2m$ marks, is

a) $2m+1$ b) $\frac{1}{3}(m+1)(2m^2+m+1)$

c) $\frac{1}{3}(m+1)(2m^2+4m+3)$ d) None of the above

143. A father with 8 children takes them 3 at a time to the zoological gardens, as often as he can without taking the same 3 children together more than once. The number of times he will go the garden, is

a) 336 b) 112 c) 56 d) None of these

144. A question paper is divided into two parts A and B and each part contain 5 questions. The number of ways in which a candidate can answer 6 questions selecting at least two questions from each part is

a) 80 b) 100 c) 200 d) None of these

145. Number if divisors of the form $(4n+2), n \geq 0$ of the integer 240 is

a) 4 b) 8 c) 10 d) 3

146. The number of ways that 8 beads of different colours be strung as a necklace is

a) 2520 b) 2880 c) 5040 d) 4320

147. The number of arrangements of the letters of the word BANANA in which the two N's do not appear adjacently, is

a) 40 b) 60 c) 80 d) 100

148. The ten's digit in $1!+4!+7!+10!+12!+13!+15!+16!+17!$ is divisible by

a) 4 b) 3! c) 5 d) 7

149. The number of ways in which a pack of 52 cards be divided equally amongst four players in order is

a) ${}^{52}C_{13}$ b) ${}^{52}C_4$ c) $\frac{52\,!}{(13\,!)^4}$ d) $\frac{52\,!}{(13\,!)^4\,4\,!}$

150. The sides AB, BC, CA of a triangle ABC have 3, 4 and 5 interior points respectively on them. The total number of triangles that can be constructed by using these points as vertices is

a) 220 b) 204 c) 205 d) 195

151. ${}^nP_r = 3024$ and ${}^nC_r = 126$, then r is

a) 5 b) 4 c) 3 d) 2

152. The value of ${}^{35}C_8 + \sum_{r=1}^{7} {}^{42-r}C_7 + \sum_{s=1}^{5} {}^{47-s}C_{40-s}$, is

a) ${}^{46}C_7$ b) ${}^{46}C_8$ c) ${}^{47}C_7$ d) ${}^{47}C_8$

153. In Q.65, the number of ways in which A_1 and A_2 are next to each other is

a) 9 ! b) 2 (9 !) c) $\frac{1}{2}(9\,!)$ d) None of these

154. The number of arrangements which can be made using all the letters of the word *LAUGH*, if the vowels are adjacent, is

a) 10 b) 24 c) 48 d) 120

155. How many ways are three to arrange the letters in the word 'GARDEN' with the vowels in alphabetical order?

a) 120 b) 240 c) 360 d) 480

156. 7 relatives of a man comprise 4 ladies and 3 gentlemen his wife has also 7 relatives, 3 of them are ladies and 4 gentlemen. In how many ways can they invite a dinner party of 3 ladies and 3 gentlemen so that there are 3 of man's relative and 3 of the wife's relative?

a) 485 b) 500 c) 486 d) 102

157. There are n-points in a plane of which p points are collinear. How many lines can be formed from these points?

a) ${}^{n}C_2 - {}^{p}C_2 + 1$ b) ${}^{n}C_2 - {}^{p}C_2$ c) $n - {}^{p}C_2$ d) ${}^{n}C_2 - {}^{p}C_2 - 1$

158. How many numbers between 5000 and 10,000 can be formed using the digits 1,2,3,4,5,6,7,8,9, each digit appearing not more than once in each number?

a) $5 \times {}^{8}P_3$ b) $5 \times {}^{8}C_8$ c) $5! \times {}^{8}C_3$ d) $5! \times {}^{8}C_3$

159. The number of ways in which 20 one rupee coins can be distributed among 5 people such that each person, gets at least 3 rupees, is

a) 26 b) 63 c) 125 d) None of these

160. The maximum number of points of intersection of 6 circles is

a) 25 b) 24 c) 50 d) 30

161. The number of all five digit numbers which are divisible by 4 that can be formed from the digits 0,1,2,3,4 (without repetition) is

a) 36 b) 30 c) 34 d) None of these

162. The total number of ways in which 4 boys and 4 girls can form a line, with boys and girls alternating, is

a) $(4!)^2$ b) $8!$ c) $2(4!)^2$ d) $4! \cdot {}^{5}P_4$

163. The products of any r consecutive natural numbers is always divisible by

a) $r!$ b) r^2 c) r^n d) None of these

164. A committee of 12 is to be formed from 9 women and 8 men in which at least 5 women have to be included in a committee. Then the number of committees in which the women are in majority and men are in majority are respectively

a) 4784, 1008 b) 2702, 3360 c) 6062, 2702 d) 2702, 1008

165. How many numbers divisible by 5 and lying between 3000 and 4000 that can be formed from the digits 1, 2, 3, 4, 5, 6 (repetition of digits is not allowed)?

a) ${}^{6}P_2$ b) ${}^{5}P_2$ c) ${}^{4}P_2$ d) ${}^{6}P_3$

166. The total number of ways of arranging the letters $AAAA\ BBB\ CC\ D\ E\ F$ in a row such that letters C are separated from one another is

a) 2772000 b) 1386000 c) 4158000 d) None of these

167. Total number of four digit odd numbers that can be formed by using 0,1,2,3,5,7 is

a) 216 b) 375 c) 400 d) 720

168. If ${}^{12}P_r = 1320$, then r is equal to

a) 5 b) 4 c) 3 d) 2

169. The lock of a safe consists of five discs each of which features the digits 0, 1, 2, ... ,9.The safe can be opened by dialing a special combination of the digits. The number of days sufficient enough to open the safe. If the work day lasts 13 h and 5 s are needed to dial one combination of digits is

a) 9 b) 10 c) 11 d) 12

170. The number of ways in which 6 rings can be worn on four fingers of one hand, is

a) 4^6 b) ${}^{6}C_4$ c) 6^4 d) 24

171. The number of integers which lie between 1 and 10^6 and which have the sum of the digits equal to 12, is

a) 8550 b) 5382 c) 6062 d) 8055

172. There are n-points $(n > 2)$ in each of two parallel lines. Every point on one line is joined to every point on the other line by a line segment drawn within the lines. The number of points (between the lines) in which these segments intersect is

a) ${}^{2n}C_2 - 2 \cdot {}^{n}C_1 + 2$ b) ${}^{2n}C_2 - 2 \times {}^{n}C_2$ c) ${}^{n}C_2 \times {}^{n}C_2$ d) None of these

173. The number of ways in which mn students can be distributed equal among n sections, is

a) $(mn)^n$ b) $\frac{(mn)!}{(m!)^n}$ c) $\frac{(mn)!}{m!}$ d) $\frac{(mn)!}{m!\,n!}$

174. There were two women participating in a chess tournament. Every participant played two games with the other participants. The number of games that the men played between themselves proved to exceed by 66 the number of games that the men played with the women. The number of participants is

a) 6 b) 11 c) 13 d) None of these

175. 20 persons are invited for a party. In how many different ways can they and the host be seated at circular table, if the two particular persons are to be seated on either side of the host?

a) 20! b) $2! \times 18!$ c) 18! d) None of these

176. Everybody in a room shakes hands with everybody else. The total number of hand shakes is 66. The total number of persons in the room is

a) 9 b) 12 c) 10 d) 14

177. The number of different words that can be formed from the letters of the word 'PENCIL' so that no two vowels are together, is

a) 120 b) 260 c) 240 d) 480

178. Consider the fourteen lines in the plane given by $y = x + r, y = x + r$, where $r \in \{0,1,2,3,4,5,6\}$. The number of squares formed by these lines whose diagonals are of length 2 is

a) 9 b) 16 c) 25 d) 36

179. Let A be a set containing 10 distinct elements. Then, the total number of distinct functions from A to A is

a) 10! b) 10^{10} c) 2^{10} d) $2^{10} - 1$

180. In a football championship, there were played 153 matches. Every team played one match with each other. The number of teams participating in the championship is

a) 17 b) 18 c) 9 d) 13

181. In how many ways can 15 members of a council sit along a circular table, when the Secretary is to sit on one side of the chairman and the Deputy Secretary on the other side?

a) $2 \times 12!$ b) 24 c) $2 \times 15!$ d) None of these

182. If in a chess tournament each contestant plays once against each of the other and in all 45 games are played, then the number of participants is

a) 9 b) 10 c) 15 d) None of these

183. These are 12 volleyball players in a college, out of which a team of 9 players is to be formed. If the captain always remains the same, then in how many ways can the team be formed?

a) 36 b) 108 c) 99 d) 165

184. In how many ways can 5 red and 4 white balls be drawn from a bag containing 10 red and 8 white balls

a) ${}^{8}C_5 \times {}^{10}C_4$ b) ${}^{10}C_5 \times {}^{8}C_4$ c) ${}^{18}C_9$ d) None of these

185. Five digited numbers with distinct digits are formed by using the digits, 5, 4, 3, 2, 1, 0. The number of those numbers which are multiples of 3, is

a) 720 b) 240 c) 216 d) 120

186. Consider the following statements :

1.These are 12 points in a plane of which only 5 are collinear, then the number of straight lines obtained by joining these points in pairs is ${}^{12}C_2 - {}^{5}C_2$

2.${}^{n+1}C_r - {}^{n-1}C_{r-1} = {}^{n}C_r + {}^{n}C_{r-2}$

3.Three letters can be posted in five letter boxes in 3^5ways.

Which of the statements given above is/are correct?

a) Only (1) b) Only (2) c) Only(3) d) None of these

187. A father with 8 children takes 3 at a time to the Zoological Gardens, as often as he can without taking the same 3 children together more than once. The number of times each child will go to the garden is

a) 56 b) 21 c) 112 d) None of these

188. The sum of all that can be formed with the digits 2,3,4,5 taken all at a time is

a) 93324 b) 66666 c) 84844 d) None of these

189. The number of ways in which 52 cards can be divided into 4 sets, three of them having 17 cards each and the fourth one having just one card

a) $\frac{52\,!}{(17\,!)^3}$ b) $\frac{52!}{(17\,!)^3\,3\,!}$ c) $\frac{51\,!}{(17\,!)^3}$ d) $\frac{51\,!}{(17\,!)^3\,3\,!}$

190. A committee of 5 is to be formed from 9 ladies and 8 men. If the committee commands a lady majority, then the number of ways this can be done is

a) 2352 b) 1008 c) 3360 d) 3486

191. The number of straight lines can be formed out of 10 points of which 7 are collinear

a) 26 b) 21 c) 25 d) None of these

192. If x, y and r are positive integers, then ${}^xC_r + {}^xC_{r-1}\,{}^yC_1 + {}^xC_{r-2}\,{}^yC_2 + \dots + {}^yC_r =$

a) $\frac{x!\,y!}{r!}$ b) $\frac{(x+y)!}{r!}$ c) ${}^{x+y}C_r$ d) ${}^{xy}C_r$

193. The greatest possible number of points of intersection of 8 straight lines and 4 circle is

a) 32 b) 64 c) 76 d) 104

194. If ${}^{16}C_r = {}^{16}C_{r+1}$, then the value of ${}^rP_{r-3}$ is

a) 31 b) 120 c) 210 d) None of these

195. $\sum_{r=0}^{m} {}^{n+r}C_n$ is equal to

a) ${}^{n+m+1}C_{n+1}$ b) ${}^{n+m+2}C_n$ c) ${}^{n+m+3}C_{n-1}$ d) None of these

196. The value of ${}^nP_r$ is equal to

a) ${}^{n-1}P_r + r.\,{}^{n-1}P_{r-1}$ b) $n.{}^{n-1}P_r + {}^{n-1}P_{r-1}$

c) $n({}^{n-1}P_r + {}^{n-1}P_{r-1})$ d) ${}^{n-1}P_{r-1} + {}^{n-1}P_r$

197. The number of ways in which 6 men and 5 women can dine at a round table, if no two women are to sit together, is

a) $6! \times 5!$ b) 30 c) $5! \times 4!$ d) $7! \times 5!$

198. The number of diagonals in a octagon will be

a) 28 b) 20 c) 10 d) 16

199. A binary sequence is an array of 0's and 1's. The number of n −digit binary sequence which contain even number of 0's is

a) 2^{n-1} b) $2^n - 1$ c) $2^{n-1} - 1$ d) 2^n

200. If ${}^{n-1}C_3 + {}^{n-1}C_4 > {}^nP_3$, then

a) $n \geq 4$ b) $n > 5$ c) $n > 7$ d) None of these

201. In a club election the number of contestants is one more than the number of maximum candidates for which a voter can vote. If the total number of ways in which a voter can vote be 126, then the number of contestants is

a) 4 b) 5 c) 6 d) 7

202. If ${}^nC_{n-r} + 3\cdot {}^nC_{n-r+1} + 3\cdot {}^nC_{n-r+2} + {}^nC_{n-r+3} = {}^xC_r$, then $x =$

a) $n+1$ b) $n+2$ c) $n+3$ d) $n+4$

203. The number of 2×2 matrices having elements 0 and 1, is

a) 8 b) 16 c) 4 d) None of these

204. If there are n number of seats and m number of people have to be seated, then how many ways are possible to do this$(m < n)$?

a) ${}^nP_m$ b) ${}^nC_m$ c) ${}^nC_n \times (m-1)!$ d) ${}^{n-1}P_{m-1}$

205. All letters of the word EAMCET are arranged in all possible ways. The number of such arrangement in which no two vowels are adjacent to each other, is

a) 360 b) 144 c) 72 d) 54

206. In how many ways 5 different beads can be arranged to form a necklace?

a) 12 b) 120 c) 60 d) 24

207. The number of permutations by taking all letters and keeping the vowels of the word COMBINE in the odd places is

a) 96 b) 144 c) 512 d) 576

208. Sixteen men compete with one another in running swimming and riding. How many prize lists could be made if there were altogether 6 prizes of different values, one for running, 2 for swimming and 3 for riding?

a) $16 \times 15 \times 14$ b) $16^3 \times 15^2 \times 14$ c) $16^3 \times 15 \times 14^2$ d) $16^2 \times 15 \times 14$

209. In how many ways can 5 boys and 5 girls sit in a circle so that no two boys sit together?

a) $5! \times 5!$ b) $4! \times 5!$ c) $\frac{5! \times 5!}{2}$ d) None of these

210. The number of diagonals that can be drawn in a polygon of 15 sides, is

a) 16 b) 60 c) 90 d) 80

211. The number of group that can be made from5 different green balls, 4 different blue balls and 3 different red balls, if at latest 1 green and 1 blue ball is to be included, is

a) 3700 b) 3720 c) 4340 d) None of these

212. The total number of words which can be formed out of the letters a, b, c, d, e, f taken 3 together, such that each word contains at least one vowel, is

a) 72 b) 48 c) 96 d) None of these

213. The number of ways in which $m + n(n \leq m + 1)$ different things can be arranged in a row such that no two of the n things may be together is

a) $\frac{(m+n)!}{m\,!\,n\,!}$ b) $\frac{m\,!\,(m+1)\,!}{(m+n)\,!}$ c) $\frac{m\,!\,(m+1)\,!}{(m-n+1)\,!}$ d) None of these

214. Number of number greater than 1000 but not greater than 4000 which can be formed with the digits 0, 1, 2, 3, 4, are

a) 350 b) 375 c) 450 d) 576

215. The number of ways in which 8 different flowers can be strung to form a garland so that 4 particular flowers are never separated is

a) $4\,! \cdot 4\,!$ b) $\frac{8\,!}{4\,!}$ c) 288 d) None of these

216. The numbers of times the digits 3 will be written when listing the integers from 1 to 1000 is

a) 269 b) 300 c) 271 d) 302

217. The number of triangles which can be formed by using the vertices of a regular polygon of $(n + 3)$ sides is 220. Then, n is equal to

a) 8 b) 9 c) 10 d) 11

218. The number of ways in which 5 ladies and 7 gentlemen can be seated in a round table so that no two ladies sit together, is

a) $\frac{7}{2}(720)^2$ b) $7(360)^2$ c) $7(720)^2$ d) 720

219. How many numbers lying between 999 and 10000 can be formed with the help of the digits 0, 2, 3, 6, 7, 8 when the digits are not be repeated?

a) 100 b) 200 c) 300 d) 400

220. The sum of the digits in the unit place of all numbers formed with the help of 3, 4, 5, 6 taken al, at a time, is

a) 18 b) 108 c) 432 d) 144

221. There are n straight lines in a plane, no two of which are parallel and no three pass through the same point. Their points of intersection are joined. Then, the number of fresh lines thus obtained is

a) $\frac{n(n-1)(n-2)}{8}$

b) $\frac{n(n-1)(n-2)(n-3)}{6}$

c) $\frac{n(n-1)(n-2)(n-3)}{8}$

d) None of the above

222. A total number of wards which can be formed out the letters a, b, c, d, e, f taken 3 together such that each word contains at least one vowel, is

a) 72 b) 48 c) 96 d) None of these

223. The number of positive odd divisors of 216 is

a) 4 b) 6 c) 8 d) 12

224. The exponent of 3 in 100 !, is

a) 33 b) 44 c) 48 d) 52

225. How many numbers lying between 10 and 1000 can be formed from the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 (repetition of digits is allowed)?

a) 1024　　b) 810　　c) 2346　　d) None of these

226. If ${}^{56}P_{r+6}$: ${}^{54}P_{r+3} = 30800 : 1$, then the value of r is

a) 40　　b) 41　　c) 42　　d) None of these

227. How many numbers greater than 24000 can be formed by using the digits 1,2,3,4,5 when no digit is repeated , is

a) 36　　b) 60　　c) 84　　d) 120

228. If 7 points out of 12 are in the same straight line, then the number of triangles formed is

a) 19　　b) 158　　c) 185　　d) 201

229. The sum of all five digit numbers that can be formed using the digits 1, 2, 3, 4, 5 when repetition of digits is not allowed, is

a) 366000　　b) 660000　　c) 360000　　d) 3999960

230. Eight different letters of an alphabet are given. Words of four letters from these are formed. The number of such words with at least one letter repeated is

a) $\binom{8}{4} - {}^8P_4$　　b) $8^4 + \binom{8}{4}$　　c) $8^4 - {}^8P_4$　　d) $8^4 - \binom{8}{4}$

231. The number of signals that can be sent using 5 flags of different colours, taking one or more at a time, is

a) 300　　b) 225　　c) 450　　d) 325

232. The number of different permutations of the word 'BANANA' is

a) 6　　b) 36　　c) 30　　d) 60

233. The number of ways in which a team of eleven players can be selected from 22 players including 2 of them and excluding 4 of them is

a) ${}^{16}C_{11}$　　b) ${}^{16}C_5$　　c) ${}^{16}C_9$　　d) ${}^{20}C_9$

234. The number of permutations of the letters of the word 'CONSEQUENCE' in which all the three E's are together, is

a) $9!\,3!$　　b) $\frac{9!}{2!\,2!}$　　c) $\frac{9!}{2!\,2!\,3!}$　　d) $\frac{9!}{2!\,3!}$

235. Sita has 5 coins each of the different denomination. The number different sums of money she can form is

a) 32　　b) 25　　c) 31　　d) None of these

236. The number of ways of dividing 52 cards amongst four players so that three players have 17 cards each and the fourth players just one card, is

a) $\frac{52!}{(17!)^3}$　　b) $52!$　　c) $\frac{52!}{17!}$　　d) None of these

237. The total number of seven-digit numbers the sum of whose digits is even is

a) 9000000　　b) 4500000　　c) 8100000　　d) None of these

238. How many different committees of 5 can be formed from 6 men and 4 women on which exact 3 men and 2 women serve?

a) 6　　b) 20　　c) 60　　d) 120

239. The number of ways can 10 letters be placed in 10 marked envelopes, so that no letter is in the right envelope are

a) $10!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\ldots+\frac{1}{10!}\right)$　　b) $10!\left(1+\frac{1}{1!}-\frac{1}{2!}+\frac{1}{3!}-\ldots-\frac{1}{10!}\right)$

c) $\left\{1+\frac{1}{1!}-\frac{1}{2!}+\frac{1}{3!}-\ldots-\frac{1}{10!}\right\}$　　d) $9!\left\{1+\frac{1}{1!}-\frac{1}{2!}+\frac{1}{3!}-\ldots-\frac{1}{10!}\right\}$

240. If the letters of the word KRISNA are arranged in all possible ways and these words are written out as in a dictionary, then the rank of the word KRISNA is

a) 324　　b) 341　　c) 359　　d) None of these

241. The number of all possible words that can be formed using the letters of the word "MATHEMATICS" is

a) $\frac{11!}{2!\,2!\,2!}$　　b) $11!$　　c) ${}^{11}C_1$　　d) None of these

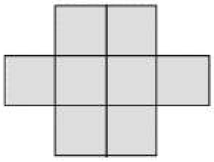
242. let P_m stand for ${}^mP_m$. Then, $1 + P_1 + 2P_2 + 3P_3 + \cdots + n \cdot P_n$ is equal to

a) $(n-1)!$　　b) $n!$　　c) $(n+1)! - 1$　　d) None of these

243. A polygon has 54 diagonals. Number of sides of this polygon is

a) 12 b) 15 c) 16 d) 9

244. Six X's have to be placed in the square of the figure such that each row contains at least one 'X'. In how many different ways can this be done?



a) 28 b) 27 c) 26 d) None of these

245. The total number of ways of dividing mn things into n equal groups, is

a) $\frac{(m\,n)\,!}{m\,!\,n\,!}$ b) $\frac{(m\,n)\,!}{(n)^m\,m\,!}$ c) $\frac{(m\,n)\,!}{(m\,!)^n\,n\,!}$ d) None of these

246. 20 persons are invited for a party. In how many different ways can they and the host be seated at circular table, if the two particular persons are to be seated on either side of the host?

a) 20! b) 218! c) 18! d) None of these

247. If ${}^{n-1}C_3 + {}^{n-1}C_4 > {}^{n}C_3$, then n is just greater than integer

a) 5 b) 6 c) 4 d) 7

248. If m and n are positive integers more than or equal to 2, $m > n$, then $(mn)\,!$ is divisible by

a) $(m\,!)^n, (n\,!)^m$ and $(m+n)\,!$ but not by $(m-n)\,!$

b) $(m+n)\,!, (m-n)\,!, (m\,!)$" but not by $(n\,!)^m$

c) $(m\,!)^n, (n\,!)^m, (m+n)\,!$ and $(m-n)\,!$

d) $(m\,!)^n$ and $(n\,!)^m$ but not by $(m+n)\,!$ and $(m-n)\,!$

249. A set contains $(2n+1)$ elements. The number of subsets of this set containing more than n elements is equal to

a) 2^{n-1} b) 2^n c) 2^{n+1} d) 2^{2n}

250. At an election there are five candidates and three members to be elected, and an elector may vote for any number of candidates not greater than the number to be elected. Then the number of ways in which an elector may vote is

a) 25 b) 30 c) 32 d) None of these

251. The total number of arrangements of the letters in the expression $a^3b^2c^4$ when written at full length, is

a) 1260 b) 2520 c) 610 d) None of these

252. The number of subsets of $\{1, 2, 3, ..., 9\}$ containing at least one odd number is

a) 324 b) 396 c) 496 d) 512

253. The number of ways in which 21 objects can be grouped into three groups of 8,7, and 6 objects is

a) $\frac{20\,!}{8\,!+7\,!+6\,!}$ b) $\frac{21\,!}{8\,!\,7\,!}$ c) $\frac{21\,!}{8\,!\,7\,!\,6\,!}$ d) $\frac{21\,!}{8\,!+7\,!+6\,!}$

254. The number of ways choosing a committee of 4 woman and 5 men from 10 women and 9 men, if Mr. A refuses to serve on the committee when Ms. B is a member of the committee, is

a) 20580 b) 21000 c) 21580 d) All the above

255. Consider the following statements :

1.The product of r consecutive natural numbers is always divisible by r.

2.The total number of proper positive divisors of 115500 is 94

3. A pack of 52 cards can be divided equally among four players order in $\frac{52!}{(13!)^4}$ ways.

Which of the statement given above is/are correct?

a) Only (1) b) Only (2) c) Only (3) d) All of (1), (2) and (3)

256. How many numbers greater than 40000 can be formed from the digits 2, 4, 5, 5, 7?

a) 12 b) 24 c) 36 d) 48

257. There are n different books and p copies of each. The number of ways in which a selection can be made from them is

a) n^p b) p^n c) $(p+1)^n - 1$ d) $(n+1)^p - 1$

258. In how many ways can 5 boys and 5 girls sit in a circle so that no two boys sit together?
a) $5! \times 5!$ b) $4! \times 5!$ c) $\frac{5! \times 5!}{2}$ d) None of these

259. The letters of the word MODESTY are written in all possible orders and these words are written out as in a dictionary, then the rank of the word MODESTY is
a) 5040 b) 720 c) 1681 d) 2520

260. If all permutations of the letters of the word AGAIN are arranged as in dictionary, then fifteen word is
a) NAAGI b) NAGAI c) NAAIG d) NAIAG

261. A shopkeeper sells three varieties of perfumes and he has a large number of bottles of the same size of each variety in his stock. There are 5 places in a row in his showcase. The number of different ways of displaying the three varieties of perfumes in the show case is
a) 6
b) 50
c) 150
d) None of these

262. The total number of ways in which six '$+$' and four '$-$' signs can be arranged in a line such that no two '$-$' signs occur together is
a) 35 b) 15 c) 30 d) None of these

263. In an examination of 9 papers a candidate has to pass in more papers, then the number of papers in which he fails in order to be successful. The number of ways in which he can be unsuccessful, is
a) 255 b) 256 c) 193 d) 319

264. The number of 5 digits telephone number having at least one of their digits repeated, is
a) 90000 b) 100000 c) 30240 d) 69760

265. If the permutations of a, b, c, d, e taken all together be written down in alphabetical order as in dictionary and numbered, then the rank of the permutation $debac$ is
a) 90 b) 91 c) 92 d) 93

266. On the occasion of Diwali festival each student of a class sends greeting cards to the others. If there are 20 students in the lass, then the total number of greeting cards exchanged by the students is
a) ${}^{20}C_2$ b) $2 \cdot {}^{20}C_2$ c) $2 \times {}^{20}P_2$ d) None of these

267. There are five different green dyes, four different blue dyes and three different red dyes. The total number of combinations of dyes that can be chosen taking at least one green and one blue dye is
a) 3255 b) 2^{12} c) 3720 d) None of these

268. The maximum number of points into which 4 circles and 4 straight lines intersect is
a) 26 b) 50 c) 56 d) 72

269. If n is even and ${}^nC_0 < {}^nC_1 < {}^nC_2 < \cdots < {}^nC_r > {}^nC_{r+1} > {}^nC_{r+2} > \cdots > {}^nC_n$, then, $r =$
a) $\frac{n}{2}$ b) $\frac{n-1}{2}$ c) $\frac{n-2}{2}$ d) $\frac{n+2}{2}$

270. Four couples (husband and wife) decide to form a committee of four members. The number of different committees that can be formed in which no couple finds a place is
a) 10 b) 12 c) 14 d) 16

271. Eight chairs are numbered 1 to 8. Two women and three men wish to occupy one chair each. First the women choose the chairs from amongst the chairs marked 1 to 4 and, then men select the chairs from amongst the remaining. The number of possible arrangements is
a) ${}^6C_3 \times {}^4C_2$ b) ${}^4C_2 \times {}^4C_3$ c) ${}^4P_2 \times {}^4P_3$ d) None of these

272. If a polygon has 44 diagonals, then the number of its sides are
a) 11 b) 7 c) 8 d) None of these

273. The number of permutations of all the letters of the word '$EXERCISES$' is
a) 60480 b) 30240 c) 10080 d) None of these

274. Let $f: \{1,2,3,4,5\} \to \{1,2,3,4,4,5\}$ that are onto and $f(x) \neq i$ is equal to

a) 9 b) 44 c) 16 d) None of these

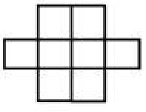
275. Ten different letters of an alphabet are given. Words with five letters are formed from these given letters. Then the number of words which have at least one letter repeated, is
a) 69760 b) 30240 c) 99748 d) None of these

276. In how many ways n books can be arranged in a row so that two specified books are not together?
a) $n! - (n-2)!$ b) $(n-1)!\,(n-2)$ c) $n! - 2(n-1)$ d) $(n-2)n!$

277. The total numbers of greater than 100 and divisible by 5, that can be formed from the digits 3,4,5,6 if no digit is repeated is
a) 24 b) 48 c) 30 d) 12

278. If the letters of the word $LATE$ be permuted and the words so formed be arranged as in a dictionary. Then, the rank of $LATE$ is
a) 12 b) 13 c) 14 d) 15

279. Six x have to be placed in the square of the figure given, such that each row contains at least one x, this can be done in



a) 24 ways b) 28 ways c) 26 ways d) 36 ways

280. Three straight lines L_1, L_2, L_3 are parallel and lie in the same plane. A total of m points are taken on L_1, n points on L_2, k points on L_3. The maximum number of triangles formed with vertices at these points are
a) ${}^{m+n+k}C_3$ b) ${}^{m+n+k}C_3 - {}^{m}C_3 - {}^{n}C_3$
c) ${}^{m+n+k}C_3 + {}^{m}C_3 + {}^{n}C_3$ d) None of the above

281. All the words that can be formed using alphabets A, H, L, U, R are written as in a dictionary (no alphabet is replaced). Then, the rank of the word RAHUL is
a) 70 b) 71 c) 72 d) 74

282. The number of natural numbers smaller than 10^4, in the decimal notation of which all the digit are different is
a) 5274 b) 5265 c) 4676 d) None of these

283. A code word consists of three letters of the English alphabet followed by two digits of the decimal system. If neither letter nor digit is repeated in any code word, then the total number of code words is
a) 1404000 b) 16848000 c) 2808000 d) None of these

284. The number of 5 digits numbers of the from $abcba$ in which $a < b$, is
a) 320 b) 340 c) 360 d) 380

285. If eleven member of a committee sit at a round table so that the President and Secretary always sit together, then the number of arrangements is
a) $10! \times 2$ b) $10!$ c) $9! \times 2$ d) None of these

286. The number of numbers that can be formed by using digits 1,2,3,4,3,2,1 so that odd digits always occupy odd places
a) $3!\,4!$ b) 34 c) 18 d) 12

287. If the letters of the word MOTHER are written in all possible orders and these words are written out as in a dictionary then the rank of the word MOTHER is
a) 240 b) 261 c) 308 d) 309

288. Consider the following statement:
1.The number of ways of arranging m different things taken all at a time in which $p \le m$ perticular things are never together is $m! - (m-p+1)!\,p!$
2. A pack of 52 cards can be divided equally among four players in order in $\frac{52!}{(13!)^4}$ ways
Which of these is/are correct?
a) Only (1) b) Only (2) c) Both of these d) None of these

289. If N is the number of positive integral solution of $x_1x_2x_3x_4 = 770$, then the value of N is

a) 250 b) 252 c) 254 d) 256

290. If a man and his wife enter in a bus, in which five seats are vacant, then the number of different ways in which they can be seated, is
a) 2 b) 5 c) 20 d) 40

291. A lady gives a dinner party to 5 guests to be selected from nine friends. The number of ways of forming the party of 5, given that two of the friends will not attend the party together is
a) 56 b) 126 c) 91 d) None of these

292. These are n distinct points on the circumference of a circle. The number of pentagons that can be formed with these points as vertices is equal to the number of possible triangles. Then, the value of n is
a) 7 b) 8 c) 15 d) 30

293. Four dice are rolled. The number of possible outcomes in which at least one dice shows 2 is
a) 625 b) 671 c) 1023 d) 1296

294. From 12 books, the difference between number of ways a selection of 5 books when one specified book is always excluded and one specified book is always included, is
a) 64 b) 118 c) 132 d) 330

295. There are n different books and m copies of each in a college library. The number of ways in which a student can make a selection of one or more books is
a) $(m+1)^n$ b) $\frac{(mn)!}{(m!)^n}$ c) ${}^{mn}C_n \times {}^nC_1$ d) $(m+1)^n - 1$

296. The number of words which can be made out of the letters of the word "MOBILE" when consonants always occupy odd places, is
a) 20 b) 36 c) 30 d) 720

297. There are n seats round a table numbered 1,2,3, ... , n. The number of ways in which $m(\leq n)$ persons can take seat is
a) ${}^nC_m$ b) ${}^nC_m \times m\,!$ c) $(m-1)\,!$ d) $(m-1)\,! \times (n-1)\,!$

298. The maximum number of points of intersection of 8 circles is
a) 16 b) 24 c) 28 d) 56

299. A lady gives a dinner party for six guests. The number of ways in which they may be selected from among ten friends, if two of the friends will not attent the party together, is
a) 112 b) 140 c) 164 d) None of these

300. The total number of arrangements which can be made out of the letters of the word 'Algebra', without altering the relative position of vowels and consonants is
a) $\frac{7\,!}{2\,!}$ b) $\frac{7\,!}{2\,!\,5\,!}$ c) $4\,!\,3\,!$ d) $\frac{4\,!\,3\,!}{2}$

301. There are 10 true-false questions in an examination. Then, these questions can be answered in
a) 240 ways b) 20 ways c) 1024 ways d) 100 ways

302. The number of ways of distributing 8 identical balls in 3 distinct boxes, so that none of the boxes is empty, is
a) 5 b) 21 c) 3^8 d) ${}^8C_3$

303. All possible two-factor products are formed from the no numbers 1,2, ... , 100. The number of factors out of the total obtained which are multiple of 3 is
a) 2211 b) 4950 c) 2739 d) None of these

304. The number of all possible selections of one or more questions from 10 given questions, each question having an alternative is
a) 3^{10} b) $2^{10}-1$ c) $3^{10}-1$ d) 2^{10}

305. In how many ways can 5 keys be put in a ring?
a) $\frac{1}{2}4!$ b) $\frac{1}{2}5!$ c) 4! d) 5!

306. There are four balls of different colours and four boxes of colours same as those of the balls. The number of ways in which the balls, one in each box, could be placed such that a ball does not go to box of its own colour, is

a) 8 b) 7 c) 9 d) None of these

307. In an steamer there are stalls for 12 animals and there are hours, cows and calves (not less then 12 each) ready to be shipped in how many ways can the ship load be made?

a) $3^{12}-1$ b) 3^{12} c) $(12)^3-1$ d) $(12)^3$

308. The number of ways to arrange the letters of the word CHEESE are

a) 120 b) 240 c) 720 d) 6

309. The number of ways in which 5 boys and 3 girls can be seated in a row so that each girl is between two boys is

a) 2880 b) 1880 c) 3800 d) 2800

310. There are 6 letters and 3 post-boxes. The number of ways in which these letters can be posted, is

a) 6^3 b) 3^6 c) ${}^6C_3$ d) ${}^6P_3$

311. The packs of 52 cards are shuffled together. The number of ways in which a man can be dealt 26 cards so that he does not get two cards of the same suit and same denomination, is

a) ${}^{52}C_{26}.2^{26}$ b) ${}^{104}C_{26}$ c) $2.{}^{52}C_{26}$ d) None of these

312. The number of proper divisors of 38808 is

a) 70 b) 72 c) 71 d) None of these

313. If ${}^8C_r - {}^7C_3 = {}^7C_2$, then r is equal to

a) 3 b) 4 c) 8 d) 6

314. The number of ways in which four persons be seated at a round table, so that all shall not have the same neighbours in any two arrangements is

a) 24 b) 6 c) 3 d) 4

315. A three digit number n is such that the last two digits of it are equal and differ from the first. The number of such n's is

a) 64 b) 72 c) 81 d) 900

316. Ten persons, amongst whom are A, B and C to speak at a function. The number of ways in which it can be done, if A wants to speak before B and B wants to speak before C is

a) $\frac{10!}{6}$ b) 3! 7! c) ${}^{10}P_3.7!$ d) None of these

317. There are 15 persons in a party and each person shakes hand with another. The total number of hand shakes is

a) ${}^{15}P_2$ b) ${}^{15}C_2$ c) 15 ! d) 2 × 15 !

318. A student is allowed to select at most n books from a collection of $(2n+1)$ books. If the total number of ways in which he can select one book is 63, then the value of n is equal to

a) 2 b) 3 c) 4 d) 1

319. There are 5 historical moments, 6 gardens and 7 shopping malls in the city. In how many ways a tourist can visit the city, if he visits at least one shopping mall?

a) $2^5.2^6.(2^7-1)$ b) $2^4.2^6.(2^7-1)$ c) $2^5.2^5.(2^6-1)$ d) None of these

320. If a, b, c, d, e are prime integers, then the number of divisors of ab^2c^2de excluding 1 as a factor is

a) 94 b) 72 c) 36 d) 71

321. The total number of permutations of $n(>1)$ different things taken not more than r at a time, when each thing may be repeated any number of times is

a) $\frac{n(n^n-1)}{n-1}$ b) $\frac{n^r-1}{n-1}$ c) $\frac{n(n^r-1)}{n-1}$ d) None of these

322. The number of ways four boys can be seated around a round table in four chairs of different colours is

a) 24 b) 12 c) 23 d) 64

323. If $a_n = \sum_{r=0}^{m} \frac{1}{{}^nC_r}$, then $\sum_{r=0}^{n} \frac{r}{{}^nC_r}$ equals

a) $(n-1)a_n$ b) na_n c) $\frac{1}{2}na_n$ d) None of these

324. How many numbers consisting of 5 digits can be formed in which the digits 3, 4 and 7 are used only once and the digit 5 is used twice?
a) 30 b) 60 c) 45 d) 90

325. All letters of the word AGAIN are permuted in all possible ways and the words so formed (with or without meaning) are written as in dictionary, then the 50th word is
a) NAAGI b) IAANG c) NAAIG d) INAGA

326. The number of seven digit integers, with sum of the digits equal to 10 and formed by using the digits 1, 2 and 3 only, is
a) 55 b) 66 c) 77 d) 88

327. If a polygon of n sided has 275 diaginals, then n is equal to
a) 25 b) 35 c) 20 d) 15

328. In how many different ways can the letters of the word 'MATCHEMATICS' be arranged?
a) 11! b) 11!/2! c) $11!/(2!)^2$ d) 11! (2!)

329. In how many ways can 21 English and 19 Hindi books be placed in a row so that no two Hindi books are together?
a) 1540 b) 1450 c) 1504 d) 1405

330. Out of 6 boys and 4 girls, a group of 7 is to be formed. In how many ways can this be done, if the group is to have a majority of boys?
a) 120 b) 80 c) 90 d) 100

331. If ${}^nC_r = {}^nC_{r-1}$ and ${}^nP_r = {}^nP_{r+1}$, then the value of n is
a) 3 b) 4 c) 2 d) 5

332. A rectangle with sides $2m-1$ and $2n-1$ divided into squere of unit length. The number of rectangle which can be formed with sides of odd length is
a) m^2n^2 b) $mn(m+1)(n+1)$ c) 4^{m+n-1} d) None of these

333. How many words can be made from the letters of the word DELHI, if L comes in the middle of every word?
a) 12 b) 24 c) 60 d) 6

334. The total number of ways in which 12 persons can be divided into three group of 4 persons each is
a) $\frac{12\,!}{(3\,!)^3\,4\,!}$ b) $\frac{12\,!}{(4\,!)^3}$ c) $\frac{12\,!}{(4\,!)^3\,3\,!}$ d) $\frac{12\,!}{(3\,!)^4}$

335. In an election there are 8 candidates, out of which 5 are to be chosen. If a voter may vote for any number of candidates but not greater than the number to be choosen, then in how many ways can a voter vote?
a) 216 b) 114 c) 218 d) None of these

336. There are 10 lamps in a hall. Each one of them can be switched on independently. The number of ways in which the hall can be illuminated, is
a) 2^{10} b) 10! c) 1023 d) 10^2

337. If $S_n = \sum_{r=0}^{n} \frac{1}{{}^nC_r}$ and $t_n = \sum_{r=0}^{n} \frac{r}{{}^nC_r}$, then $\frac{t_n}{s_n}$ is equal to
a) $\frac{n}{2}$ b) $\frac{n}{2}-1$ c) $n-1$ d) $\frac{2n-1}{2}$

338. If ${}^nC_{r-1} = 36$, ${}^nC_r = 84$ and ${}^nC_{r+1} = 126$, then the value of r is
a) 1 b) 2 c) 3 d) None of these

339. The number of ways of arranging letters of the word HAVANA so that V and N do not appear together is
a) 60 b) 80 c) 100 d) 120

340. The rank of the word MOTHER when the letters of the word are arranged alphabetically as in a dictionary, is
a) 261 b) 343 c) 309 d) 273

341. The number of four-digit even numbers that can be formed using 0,1,2,3,4,5,6 without repetition is
a) 120 b) 300 c) 420 d) 20

342. ${}^nC_r + 2\,{}^nC_{r-1} + {}^nC_{r-2}$ is equal to

a) $^{n+1}C_r$ b) $^{n}C_{r+1}$ c) $^{n-1}C_{r+1}$ d) None of these

343. If $^{12}P_r = {}^{11}P_6 + 6 \cdot {}^{11}P_5$ then r is equal to

a) 6 b) 5 c) 7 d) None of these

344. A box contains two white balls, three black balls and four red balls. In how many ways can three balls be drawn from the box, if at least one black ball is to be included in the draw?

a) 64 b) 45 c) 46 d) None of these

345. There are 18 points in a plane such that no three of them are in the same line except five points which are collinear. The number of triangles formed by these points is

a) 805 b) 806 c) 816 d) None of these

346. A person wishes to make up as many different parties as he can out of his 20 friends such that each party consists of the same number of persons. The number of friends he should invite at a time, is

a) 5 b) 10 c) 8 d) None of these

347. The number of diagonals in a polygon of m sides is

a) $\frac{1}{2!}m(m-5)$ b) $\frac{1}{2!}m(m-1)$ c) $\frac{1}{2!}m(m-3)$ d) $\frac{1}{2!}m(m-2)$

348. The value of $\sum_{r=1}^{n} \frac{^nP_r}{r!}$ is

a) 2^n b) $2^n - 1$ c) 2^{n-1} d) $2^n + 1$

349. The number of ways in which any four letters can be selected from the word 'CORGOO' is

a) 15 b) 11 c) 7 d) None of these

350. The number of ways in which one can select three distinct integers between 1 and 30, both inclusive, whose sum is even, is

a) 455 b) 1575 c) 1120 d) 2030

351. In how many ways 3 letters can be posted in 4 letter-boxes, if all the letters are not posted in the same letter-box?

a) 63 b) 60 c) 77 d) 81

352. If 7 points out of 12 are in the same straight line, then the number of triangles formed is

a) 19 b) 158 c) 185 d) 201

353. A bag contains 3 black, 4 white and 2 red balls, all the balls being different. The number of selections of at most 6 balls containing balls of all the colours, is

a) 42(4!) b) $2^6 \times 4!$ c) $(2^6 - 1)(4!)$ d) None of these

354. Total number of n digit numbers $(n > 1)$ having the property that no two consecutive digits are same, is

a) 8^n

b) 9^n

c) 9.10^{n-1}

d) None of these

355. If $^{n-1}C_6 + {}^{n-1}C_7 > {}^{n}C_6$, then

a) $n > 4$ b) $n > 12$ c) $n \geq 13$ d) $n > 13$

356. If $r > p > q$, the number of different selections of $p + q$ thing taking r at a time, where p things are identical and q other things are identical, is

a) $p + q - r$ b) $p + q - r + 1$ c) $r - p - q + 1$ d) None of these

357. $S_1, S_2, \dots, S_{10}$ are the speakers in a conference. If S_1 addresses only after S_2, then the number of ways the speakers address is

a) 10! b) 9! c) $10 \times 8!$ d) $\frac{10!}{2!}$

358. 12 persons are to be arranged to a round table. If two particular persons among them are not to be side by side, the total number of arrangements is

a) 9 (10 !) b) 2 (10 !) c) 45 (8 !) d) 10 !

359. A student is to answer 10out of 13 questions in an examination such that he must choose at least 4 from the first five questions. The number of choices available to him is

a) 140 b) 196 c) 280 d) 346

360. The straight lines I_1, I_2, I_3 are parallel and lie in the same plane. A total numbers of m points are taken on I_1, n points on I_2, k points on I_3. The maximum number of triangles formed with vertices at these points is

a) ${}^{m+n+k}C_3$
b) ${}^{m+n+k}C_3 - {}^{m}C_3 - {}^{n}C_3 - {}^{k}C_3$
c) ${}^{m}C_3 + {}^{n}C_3 + {}^{k}C_3$
d) None of the above

361. In a Mathematics paper there are three sections containing 4, 5 and 6 questions respectively. From each section 3 questions are to be answered. In how many ways can be selection of questions be made?

a) 34
b) 800
c) 1600
d) 9600

362. A library has a copies of one book, b copies of each of two books, c copies of each of three books and single copies of d books. The total number of ways in which these book can be distributed, is

a) $\frac{(a+b+c+d)!}{a!\,b!\,c!}$
b) $\frac{(a+2b+3c+d)!}{a!\,(b!)^2(c!)^3}$
c) $\frac{(a+2b+3c+d)}{a!\,b!\,c!}$
d) None of these

363. If ${}^{n+2}C_8 : {}^{n-2}P_4 = \frac{57}{16}$, then n is equal to

a) 19
b) 2
c) 20
d) 5

364. If $\frac{1}{{}^4C_n} = \frac{1}{{}^5C_n} + \frac{1}{{}^6C_n}$, then n is equal to

a) 3
b) 2
c) 1
d) 0

365. If $P(n, r) = 1680$ and $C(n, r) = 70$, then $69n + r!$ is equal to

a) 128
b) 576
c) 256
d) 625

366. How many 10 digit numbers can be written by using the digits 1 and 2?

a) ${}^{10}C_1 + {}^{9}C_2$
b) 2^{10}
c) ${}^{10}C_2$
d) 10!

367. If P_m stands for ${}^mP_m$, then $1 + 1P_1 + 2P_2 + 3P_3 + \ldots + n.P_n$ is equal to

a) $n!$
b) $(n+3)!$
c) $(n+2)!$
d) $(n+1)!$

368. If $2\,{}^{n+1}P_{n-1} : {}^{2n-1}P_n = 3:5$, then n is equal to

a) 4
b) 6
c) 3
d) 8

369. How many numbers of 6 digits can be formed from the digits of the number 112233?

a) 30
b) 60
c) 90
d) 120

370. If r, s, t are prime numbers and p, q are the positive integers such that LCM of p, q is $r^2s^4t^2$, then the number of ordered pairs (p, q) is

a) 252
b) 254
c) 225
d) 224

371. The number of words which can be formed the letters of the word MAXIMUM, if two consonants cannot occur together, is

a) 4!
b) $3! \times 4!$
c) 7!
d) None of these

PERMUTATIONS AND COMBINATIONS

: ANSWER KEY :

1)	d	2)	c	3)	c	4)	c	157)	a	158)	a	159)	a	160)	d
5)	a	6)	a	7)	c	8)	c	161)	b	162)	c	163)	a	164)	d
9)	c	10)	b	11)	b	12)	b	165)	c	166)	b	167)	d	168)	c
13)	d	14)	a	15)	b	16)	c	169)	c	170)	a	171)	c	172)	c
17)	c	18)	d	19)	a	20)	a	173)	b	174)	c	175)	b	176)	b
21)	d	22)	d	23)	d	24)	a	177)	d	178)	c	179)	b	180)	b
25)	c	26)	b	27)	a	28)	a	181)	a	182)	b	183)	d	184)	b
29)	a	30)	a	31)	a	32)	a	185)	c	186)	d	187)	b	188)	a
33)	c	34)	a	35)	a	36)	a	189)	b	190)	d	191)	c	192)	c
37)	a	38)	b	39)	a	40)	a	193)	d	194)	d	195)	a	196)	a
41)	c	42)	b	43)	a	44)	c	197)	a	198)	b	199)	a	200)	c
45)	b	46)	c	47)	d	48)	a	201)	d	202)	c	203)	b	204)	a
49)	c	50)	d	51)	b	52)	d	205)	c	206)	a	207)	d	208)	b
53)	c	54)	b	55)	c	56)	a	209)	b	210)	c	211)	b	212)	c
57)	c	58)	b	59)	b	60)	c	213)	c	214)	b	215)	a	216)	b
61)	a	62)	d	63)	d	64)	b	217)	b	218)	a	219)	c	220)	b
65)	c	66)	b	67)	a	68)	c	221)	c	222)	c	223)	a	224)	c
69)	c	70)	c	71)	b	72)	b	225)	b	226)	b	227)	c	228)	c
73)	b	74)	b	75)	d	76)	a	229)	d	230)	c	231)	d	232)	d
77)	b	78)	c	79)	c	80)	d	233)	c	234)	b	235)	c	236)	a
81)	b	82)	c	83)	a	84)	a	237)	b	238)	d	239)	a	240)	a
85)	d	86)	b	87)	d	88)	b	241)	a	242)	c	243)	a	244)	c
89)	b	90)	c	91)	b	92)	a	245)	c	246)	b	247)	d	248)	c
93)	a	94)	c	95)	b	96)	c	249)	d	250)	a	251)	a	252)	c
97)	d	98)	a	99)	b	100)	b	253)	d	254)	a	255)	d	256)	d
101)	b	102)	d	103)	a	104)	a	257)	c	258)	b	259)	c	260)	c
105)	b	106)	a	107)	c	108)	b	261)	c	262)	a	263)	b	264)	d
109)	a	110)	a	111)	a	112)	c	265)	d	266)	b	267)	c	268)	b
113)	c	114)	b	115)	b	116)	b	269)	a	270)	d	271)	d	272)	a
117)	b	118)	b	119)	b	120)	a	273)	b	274)	b	275)	a	276)	b
121)	b	122)	a	123)	c	124)	b	277)	d	278)	c	279)	c	280)	d
125)	c	126)	d	127)	a	128)	a	281)	d	282)	a	283)	a	284)	c
129)	d	130)	c	131)	a	132)	d	285)	c	286)	c	287)	d	288)	c
133)	c	134)	b	135)	a	136)	a	289)	d	290)	c	291)	c	292)	b
137)	b	138)	c	139)	a	140)	b	293)	b	294)	c	295)	d	296)	b
141)	b	142)	c	143)	c	144)	c	297)	b	298)	d	299)	b	300)	d
145)	a	146)	a	147)	a	148)	b	301)	c	302)	b	303)	c	304)	c
149)	c	150)	c	151)	b	152)	d	305)	a	306)	c	307)	b	308)	a
153)	b	154)	c	155)	c	156)	a	309)	a	310)	b	311)	a	312)	a

313)	a	314)	c	315)	c	316)	a
317)	b	318)	b	319)	a	320)	d
321)	c	322)	a	323)	c	324)	b
325)	c	326)	c	327)	a	328)	d
329)	a	330)	d	331)	a	332)	a
333)	b	334)	c	335)	c	336)	c
337)	a	338)	c	339)	b	340)	c
341)	c	342)	d	343)	a	344)	a

345)	b	346)	b	347)	c	348)	b
349)	c	350)	d	351)	b	352)	c
353)	a	354)	b	355)	d	356)	b
357)	d	358)	a	359)	b	360)	b
361)	b	362)	b	363)	a	364)	b
365)	b	366)	b	367)	d	368)	a
369)	c	370)	c	371)	a		

PERMUTATIONS AND COMBINATIONS

: HINTS AND SOLUTIONS :

1 **(d)**
We have,
Required number of ways $= (2+1)(3+1)(4+10-1) = 59$

2 **(c)**
Given, $m = {}^nC_2 = \frac{n!}{2!(n-2)!} = \frac{n(n-1)}{2}$

Now, ${}^nC_2 = \frac{m!}{2!(m-2)!} = \frac{m(m-1)}{2}$

$= \frac{\frac{n(n-1)}{2}\cdot\left(\frac{n^2-n-2}{2}\right)}{2}$

$= \frac{(n+1)n(n-1)(n-2)}{8}$

$= 3.\ {}^{n+1}C_4$

3 **(c)**
Let the boxes be marked as A, B, C. We have to ensure that no box remains empty and all five balls have to put in. There will be two possibilities.

(i) Any two box containing one ball each and 3rd box containing 3 balls. Number of ways

$= A(1)B(1)C(3)$

$= {}^5C_1.{}^4C_1.{}^3C_3 = 5.4.1 = 20$

Since, the box containing 3 balls could be any of the three aboxes A, B, C. Hence, the required number of ways $20 \times 3 = 60$

(ii) Any two box containing 2 balls each and 3rd containing 1 ball, the number of ways

$= A(2)B(2)C(1) = {}^5C_2.\ {}^3C_2.\ {}^1C_1$

$= 10 \times 3 \times 1 = 30$

Since, the box containing 1 ball could be any of the three boxes A, B, C. Hence, The required number of ways

$= 30 \times 3 = 90$

Hence, total number of ways $= 60 + 90 = 150$

4 **(c)**
Let n be the number of sides of the polygon

$n.160° = (n-2).180°$

$\Rightarrow 20°.n = 360°$

$\therefore\ n = 18$

Then number of diagonals $= {}^{18}C_2 - 18 = 153 - 18 = 135$

5 **(a)**
Required number of ways $= {}^nC_m \times m! = {}^nP_m$

6 **(a)**
$\sum_{r=0}^{m} {}^{n+r}C_n = \sum_{r=0}^{m} {}^{n+r}C_r$
$= {}^nC_0 + {}^{n+1}C_1 + {}^{n+2}C_2 + \ldots + {}^{n+m}C_m$
$= {}^{n+1}C_0 + {}^{n+1}C_1 + {}^{n+2}C_2 + \ldots + {}^{n+m}C_m$
$[\because {}^{n+1}C_0 = {}^nC_0]$
$= {}^{n+2}C_1 + {}^{n+2}C_2 + \ldots + {}^{n+m}C_m$
$= {}^{n+m}C_{m-1} + {}^{n+m}C_m$
$= {}^{n+m+1}C_m \quad [\because {}^nC_{r-1} + {}^nC_r = {}^{n+r}C_r]$
$= {}^{n+m+1}C_{n+1} \quad [\because {}^nC_r = {}^nC_{n-r}]$

7 **(c)**
Required number of ways
$= {}^{12}C_4 \times {}^8C_4 \times {}^4C_4$
$= \frac{12!}{8! \times 4!} \times \frac{8!}{4! \times 4!} \times 1 = \frac{12!}{(4!)^3}$

8 **(c)**
Required number of selections
$= {}^8C_4 + {}^8C_5 + {}^8C_6 + {}^8C_7 + {}^8C_8$
$= 70 + 56 + 28 + 8 + 1 = 163$

9 **(c)**
Arrange the letter of the word COCHIN as in the order of dictionary CCHINO

Which number of words with the two C's occupying first and second place= 4!

Number of words starting with CH, CI, CN is 4! each

$\therefore$ Total number of ways $= 4! + 4! + 4! + 4! = 96$

There are 96 words before COCHIN

10 **(b)**

The villagers can go to the town in ${}^5C_1$ ways and they return back in ${}^5C_1$ ways.

$\therefore$ Total number of ways= ${}^5C_1 \times {}^5C_1 = 25$

11 **(b)**

The number of distinct n-digit numbers to be formed using digits 2, 5 and 7 is 3^n. We have to find n so that

$3^n \geq 900 \Rightarrow 3^{n-2} \geq 100$

$\Rightarrow n - 2 \geq 5 \Rightarrow n \geq 7$

So the least value of n is 7

12 **(b)**

We have,

${}^nC_{r-1} = 36,\ {}^nC_r = 84,\ {}^nC_{r+1} = 126$

$\Rightarrow \frac{{}^nC_r}{{}^nC_{r-1}} = \frac{84}{36}$ and $\frac{{}^nC_{r+1}}{{}^nC_r} = \frac{126}{84}$

$\Rightarrow \frac{n-r+1}{r} = \frac{7}{3}$ and $\frac{n-r}{r+1} = \frac{3}{2}$

$\Rightarrow 3n - 10r + 3 = 0$ and $2n - 5r - 3 = 0 \Rightarrow$ $r = 3, n = 9$

13 **(d)**

The required number is the coefficient of x^{11} in $(1 + x + x^2 + \cdots + x^{11})^6 = {}^{11+6-1}C_{6-1} = {}^{16}C_5$

14 **(a)**

Let number of sides of polygon=n

$\Rightarrow \quad {}^nC_2 - n = 44 \qquad$ [given]

$\Rightarrow \quad \frac{n!}{2!\,(n-2)!} - n = 44$

$\Rightarrow \quad n(n-1) - 2n = 88$

$\Rightarrow \quad n^2 - 3n - 88 = 0$

$\Rightarrow \quad (n-11)(n+8) = 0 \quad \Rightarrow \quad n = 11, -8$

Since, sides cannot be negative

$\therefore \quad n = 11$

15 **(b)**

12 balls can be distributed between two friends A and B in two ways

(i) Friend A receives 8 and B receives 4

(ii) Friend B receives 8 and A receives 4

$\therefore$ Required number of ways $= \frac{12!}{8!\,4!} + \frac{12!}{4!\,8!} =$ $2\left(\frac{12!}{8!\,4!}\right)$

16 **(c)**

Digit at the extreme left can be chosen by 9 ways as zero cannot be the first digit. Now for the second digit it can be done in 9 ways as consecutive digits are not same. And this is same for next digits. Hence, number of ways are $9 \times 9 \times 9 \times \ldots.. \times n$ times= 9^n

17 **(c)**

The number forms by the figure 4, 5, 6, 7, 8 which is greater than 56000 is in two cases.

Case I Let the ten thousand digit place number be greater than 5. The number of numbers

$= 3 \times 4 \times 3 \times 2 \times 1 = 72$

Case II Let the ten thousand digit number be 5 and thousand digit number be either 6 or greater than 6. Then, the number of numbers $= 3 \times 3 \times 2 \times 1 = 18$

$\therefore$ Required number of ways $= 72 + 18 = 90$

18 **(d)**

Total number of points in a plane is $3p$

$\therefore$ Maximum number of triangles

$= {}^{3p}C_3 - 3.\,{}^pC_3$

[here, we subtract those triangles which points are in a line]

$= \frac{(3p)!}{(3p-3)!\,3!} - 3.\frac{p!}{(p-3)3!}$

$= \frac{3p(3p-1)(3p-2)}{3 \times 2} - \frac{3 \times p(p-1)(p-2)}{3 \times 2}$

$= \frac{p}{2}[9p^2 - 9p + 2 - (p^2 - 3p + 2)] = p^2(4p - 3)$

19 **(a)**

$\because \quad 240 = 2^4.3.5$

$\therefore$ Total number of divisors=(4+1)(2)(2)=20

Out of these 2, 6, 10 and 30 are of the from $4n + 2$

20 **(a)**

Given word is MISSISSIPPI

Here, I=4 times, S=4 times, P=2 times, M=1 time

_M_I_I_I_I_P_P_

Required number of words $= {}^8C_4 \times \frac{7!}{4!2!}$

$= {}^8C_4 \times \frac{7 \times 6!}{4!\,2!} = 7.\,{}^8C_4.\,{}^6C_4$

21 **(d)**

If triangle is formed including point 'P' the other points must be one from l_1 and other point from l_2. Number of triangle formed with $P = {}^3C_1 \times {}^5C_1 = 15$ ways

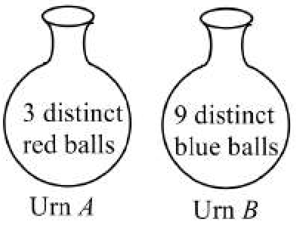
When P is not included.

Number of triangle formed

$= {}^3C_2 \times {}^5C_1 + {}^3C_1 \times {}^5C_2 = 15 + 15 = 30$

Total number of triangles=15+30=45

22 **(d)**



The number of ways in which two balls form urn A and two balls from urn B can be selected

$= {}^3C_2 \times {}^9C_2 = 3 \times 36 = 108$

23 **(d)**

The word 'ARTICLE' has 3 vowels and 4 consonants and according to problem we have to put the 3 vowels on 3 even places and 4 consonants in the remaining places.

∴ The required number of ways

$= 3! \times 4! = 6 \times 24 = 144$

24 **(a)**

$[1.3.5 \dots\dots (2n-1)]2^n$

$$= \frac{1.2.3.4.5.6 \dots\dots (2n-1)(2n)2^n}{2.4.6 \dots\dots 2n}$$

$$= \frac{(2n)!\, 2^n}{2^n(1.2.3 \dots\dots n)} = \frac{(2n)!}{n!}$$

25 **(c)**

Each letter can be posted in any one of the 7 letter boxes. So, required number of ways $= 7 \times 7 \times 7 \times 7 \times 7 = 7^5$

26 **(b)**

Since, 2 persons can drive the car, therefore we have to select 1 from these two. This can be done in ${}^2C_1$ways. Now, from the remaining 5 persons we have to select 2 which can be done in ${}^5C_2$ ways.

Therefore, the required number of ways in which the car can be filled

$= {}^5C_2 \times {}^2C_1 = 10 \times 2 = 20$

27 **(a)**

We have,

Required number of ways

= Coefficient of x^{10} in $(1 + x + x^2 + \cdots)^4$

= Coefficient of x^{10} in $(1-x)^{-4}$

$= {}^{10+4-1}C_{4-1} = {}^{13}C_3 = 286$

28 **(a)**

The required number of ways

$= {}^5C_4 \cdot {}^5C_2 + {}^5C_3 + {}^5C_2 \cdot {}^5C_4$

$= 50 + 100 + 50 = 200$

29 **(a)**

Let n be the number of terms

$\because \quad {}^nC_2 = 36$

$\Rightarrow \quad \frac{n(n-1)}{1.2} = 36$

$\Rightarrow \quad n(n-1) = 72 = 9 \times 8$

$\Rightarrow \quad n = 9$

30 **(a)**

The number formed will be divisible by 4 if the number formed by the two digits on the extreme right is divisible by 4 i.e. it should be 12,24,32,52,44

The number of numbers ending in $12 = 5 \times 5$

The number of numbers ending in $24 = 5 \times 5$

The number of numbers ending in $32 = 5 \times 5$

The number of numbers ending in $52 = 5 \times 5$

The number of numbers ending in $44 = 5 \times 5$

Thus, the required number

$= 5 \times 5 + 5 \times 5 + 5 \times 5 + 5 \times 5 + 5 \times 5 = 125$

32 **(a)**

8 different beads can be arranged in circular form in $(8-1)! = 7!$ ways. Since, there is no distinction between the clockwise and anticlockwise arrangement. So, the required number of arrangements$= \frac{7!}{2} = 2520$

33 **(c)**

Required number of arrangements

$= {}^6P_5 \times 4! = 720 \times 24 = 17280$

34 **(a)**

We have,

$${}^nC_r + 4 \cdot {}^nC_{r-1} + 6 \cdot {}^nC_{r-2} + 4 \cdot {}^nC_{r-3} + {}^nC_{r-4}$$
$$= ({}^nC_r + {}^nC_{r-1}) + 3({}^nC_{r-1} + {}^nC_{r-2}) + 3({}^nC_{r-2} + {}^nC_{r-3}) + ({}^nC_{r-3} + {}^nC_{r-4})$$
$$= {}^{n+1}C_r + 3 \cdot {}^{n+1}C_{r-1} + 3 \cdot {}^{n+1}C_{r-2} + {}^{n+1}C_{r-3}$$
$$= ({}^{n+1}C_r + {}^{n+1}C_{r-1}) + 2({}^{n+1}C_{r-1} + {}^{n+1}C_{r-2}) + ({}^{n+1}C_{r-2} + {}^{n+1}C_{r-3})$$
$$= {}^{n+2}C_r + 2 \cdot {}^{n+2}C_{r-1} + {}^{n+2}C_{r-2}$$
$$= ({}^{n+2}C_r + {}^{n+2}C_{r-1}) + ({}^{n+2}C_{r-1} + {}^{n+2}C_{r-2})$$
$$= {}^{n+3}C_r + {}^{n+3}C_{r-1} = {}^{n+4}C_r$$

35 **(a)**

There can be two types of numbers

(i) any one of the digits 1,2,3,4 repeats thrice and the remaining digits only once i.e. of the type 1,2,3,4,4,4

(ii) any two of the digits 1,2,3,4 repeat twice and the remaining two only once i.e. of the type 1,2,3,4,4

Number of numbers of the type 1 2 3 4 4 4

$= \frac{6!}{3!} \times {}^4C_1 = 480$

Number of numbers of the type 1 2 3 3 4 4

$= \frac{6!}{2!\ 2!} \times {}^4C_2 = 1080$

So, the required number $= 480 + 1080 = 1560$

36 **(a)**

First arrange m men in a row in $m!$ ways. Since, $n < m$ and no two women can sit together in any one of the $m!$ arrangement, there are $(m+1)$ places in which n women can be arranged in ${}^{m+1}P_n$ ways.

$\therefore$ The required number of arrangements of m men and n women $(n < m)$

$= m!^{m+1}P_n = \frac{m!\,(m+1)!}{(m-n+1)!}$

37 **(a)**

Given, $6 \leq a + b + c \leq 10$

$\therefore\ a + b + c = 6, 7, 8, 9, 10$

Here $a \geq 1, b \geq 1, c \geq 1$

$\therefore$ Required number of ways

$= {}^5C_2 + {}^6C_2 + {}^7C_2 + {}^8C_2 + {}^9C_2$

$= 110$

38 **(b)**

We have,

${}^{n+2}C_8 : {}^{n-2}P_4 = 57 : 16$

$\Rightarrow \frac{(n+2)!\,(n-6)!}{(n-6)!\,(n-2)!\,8!} = \frac{57}{16}$

$\Rightarrow (n+2)(n+1)n(n-1) = 143640$

$\Rightarrow (n^2+n-2)(n^2+n) = 143640$

$\Rightarrow (n^2+n)^2 - 2(n^2+n) + 1 = 143641$

$\Rightarrow (n^2+n-1)^2 = (379)^2$

$\Rightarrow n^2+n-1 = 379 \qquad [\because n^2+n-1 > 0]$

$\Rightarrow n^2+n-380 = 0$

$\Rightarrow (n+20)(n-19) = 0 \Rightarrow n = 19$ [$\because n$ is not negative]

39 **(a)**

A triangle is obtained by joining three non-collinear points. So number of triangles on joining 3 points out of 10 points $= {}^{10}C_3$. But, 6 points are collinear and on joining any three out of these 6, we do not obtain a triangle

Hence, the required number of triangles

$= {}^{10}C_3 - {}^6C_3 = 120 - 20 = 100$

40 **(a)**

$\because$ Given word is CRICKET

total number of letters are 7 out of which two letters 'C' are count as one

$\therefore$ Required number of ways of words before the word CRICKET$= 5! \times 4 + 2 \times 4! + 2!$

$= 480 + 48 + 2 = 530$

41 **(c)**

A man has two options for every friend either they invited it or not.

$\therefore$ Required number of ways $= 2^7 - 1 = 127$

[Since, we have to subtract those cases in which he does not invite any friend *ie*, ${}^nC_0 = 1$]

Alternate Solution

Required number of ways $= {}^7C_1 + {}^7C_2 + {}^7C_3 + \dots + {}^7C_7$

$= 2^7 - 1$

42 **(b)**

Required number of ways

=coefficient of x^{16}in $(x^3 + x^4 + x^5 + \dots\dots + x^{16})^4$

=coefficient of x^{16}in $x^{12}(1 + x + x^2 + \dots\dots + x^{12})^4$

=coefficient of x^4 in $(1 - x^{13})^4(1-x)^{-4}$

=coefficient of x^4 in $(1 - 13x^5 + \dots)$

$\times \left[1 + 4x + \dots\dots + \frac{(r+1)(r+2)(r+3)}{3!}x^r\right]$

$= \frac{(4+1)(4+2)(4+3)}{3!} = 35$

43 **(a)**

Since, $38808 = 2^3 \times 3^2 \times 7^2 \times 11^1$

$\therefore$ Number of divisors $= 4 \times 3 \times 3 \times 2 - 2$

$= 72 - 2 = 70$

44 **(c)**

An even number has an even digit at unit place

$\therefore$ Required number of even numbers

= Number of even numbers having 0 at unit's place
+ Number of even numbers having a non-zero digit at unit's place
$= {}^6C_3 \times 3! \times 1 + {}^3C_1({}^6C_3 \times 3! - {}^5C_2 \times 2!)$
$= 120 + 3 \times (120 - 20) = 420$

45 **(b)**

Given, ${}^nC_r = 30240$ and ${}^nC_r = 252$

$\frac{n!}{(n-r)!} = 30240$ and $\frac{n!}{(n-r)!r!} = 252$

$\Rightarrow \quad r! = \frac{30240}{252} = 120 \quad \Rightarrow \quad r = 5$

$\therefore \quad \frac{n!}{(n-5)!} = 30240$

$\Rightarrow \quad n(n-1)(n-2)(n-3)(n-4)$
$= 10(10-1)(10-2)(10-3)(10-4)$
$\Rightarrow \quad n = 10$

Hence, required ordered pair is (10, 5)

46 **(c)**

${}^nC_r + 2\,{}^nC_{r-1} + {}^nC_{r-2}$
$= {}^nC_r + {}^nC_{r-1} + {}^nC_{r-1} + {}^nC_{r-2}$

$= {}^{n+1}C_r + {}^{n+1}C_{r-1} = {}^{n+2}C_r$

47 **(d)**

4 odd digits 3,3,5,5 can occupy 4 even places in $\frac{4!}{2!2!}$ ways and 5 even digits 2,2,8,8,8 can occupy 5 odd places in $\frac{5!}{3!2!}$ ways

∴ Required number of nine digit numbers

$= \frac{4!}{2!2!} \times \frac{5!}{3!2!} = 60$

48 **(a)**

In a dictionary the words at each stage are arranged in alphabetical order. In the given problem we must therefore consider the words beginning with A, D, M, N, O, R in order. A will occur in the first place as often as there are ways of arranging the remaining 5 letters all at a time. i.e. A will occur 5! Times. D, M, N, O will occur in the first place the same number of times. So,

Number of words starting with $A = 5! = 120$
Number of words starting with $D = 5! = 120$
Number of words starting with $M = 5! = 120$
Number of words starting with $N = 5! = 120$
Number of words starting with $O = 5! = 120$
Number of words beginning with RAD or RAM, is 3!

Now the words beginning with 'RAN' must follow
First one is $RANDMO$ and the next one is $RANDOM$

∴ Rank of $RANDOM = (5!)5 + (3!)2 + 2 = 614$

49 **(c)**

The number of ways in which 4 novels can be selected
$= {}^6C_4 = 15$
The number of ways in which 1 dictionary can be selected
$= {}^3C_1 = 3$
4 novels can be arranged in 4! ways
∴ The total number of ways
$= 15 \times 4! \times 3 = 15 \times 24 \times 3 = 1080$

50 **(d)**

Required number of possible outcomes
= Total number of possible outcomes – Number of possible outcomes in which 5 does not appear on any dice
$= 6^3 - 5^3 = 216 - 125 = 91$

51 **(b)**

We have,
The required number $= {}^{3+35-1}C_{3-1} = {}^{37}C_2 = 666$

52 **(d)**

We have,
${}^{10}C_{x-1} > 2\,{}^{10}C_x$

$\Rightarrow \frac{10!}{(11-x)!(x-1)!} > 2 \cdot \frac{10!}{(10-x)!x!}$

$\Rightarrow \frac{1}{11-x} > \frac{2}{x}$

$\Rightarrow 3x > 22 \Rightarrow x > \frac{22}{3} \Rightarrow x \geq 8$

Thus, the smallest value of x satisfying the above inequality is 8

53 **(c)**

The number of ways in which 5 pictures can be hung from 7 picture nailes on the wall is same as the number of arrangements of 7 things by taking 5 at a time.

Hence, the required number $= {}^7P_5 = \frac{7!}{2!} = 2520$

54 **(b)**

Let A, B be the corresponding speakers.
Without any restriction the eight persons can be arranged among themselves in 8 ! ways; but the number of ways in which B speaks A speaks before B and the number of ways in which B speaks before A make up 8 !. Also, the number of ways in which A speaks before B is exactly same as the number of ways in which B speaks before A.

So, the required number of ways $= \frac{1}{2}(8!) = 20160$

56 **(a)**
A number is divisible by 4, if the number formed by the last two digits is divisible by 4. A four digit number divisible by 4 formed with the digits, 1,2,3,5,6 can have last two digits as follows:
× × 12
× × 16
× × 32
× × 36
× × 52
× × 56
Corresponding to each of these ways first two places can be filled in ${}^3C_2 \times 2!$ Ways
Hence, required number of numbers = ${}^3C_2 \times 2! \times 6 = 36$

57 **(c)**
Each set is having $(m+2)$ parallel lines and each parallelogram is formed by choosing two straight lines from the first set and two straight lines from the second set. Two straight lines from the first set can be chosen in ${}^{m+2}C_2$ ways and two straight lines from the second set can be chosen in ${}^9C_5$ ways. Hence, the total number of parallelograms formed
$= {}^{m+2}C_2 \cdot {}^{m+2}C_2 = ({}^{m+2}C_2)^2$

58 **(b)**
$\because 720 = 2^4 \times 3^2 \times 5^1$

$\therefore$ Sum of all odd divisors $= (1+3+3^2)(1+5^1)$

$= 13 \times 6 = 78$

59 **(b)**
Required number of ways= $6! \times 3! = 4320$

60 **(c)**
Since, 5 does not occur in 1000, we have to count the number of times 5 occurs when we list the integers from 1 to 999. Any number between 1 and 999 is of the form $xyz, 0 \le x, y, z \le 9$
The number in which 5 occurs exactly once
$= ({}^3C_1)9 \times 9 = 243$
The number in which 5 occurs exactly twice=$({}^3C_2.9) = 27$
The number in which 5 occurs in all three digits =1
Hence, the number of times 5 occurs
$= 1 \times 243 + 2 \times 27 + 3 \times 1 = 300$

61 **(a)**
Since the number of faces is same as the number of colours. Therefore, the number of ways of painting them = 1

62 **(d)**
When repetition is allowed then, number of four digits numbers that can be formed using 1, 2, 3, 4, 5= 5^4
and when repetition of digits is not allowed, then number of 4 digits numbers which can be formed is ${}^5P_4 = 5!$
$\therefore$ The number of ways in which at least one digit is repeated= $5^4 - 5!$

63 **(d)**
The number of ways of arranging 8 men= 7!
The number of ways of arranging 4 women such that no two women can sit together= ${}^8P_4$
$\therefore$ Required number of ways= $7!\,{}^8P_4$

64 **(b)**
Required number of ways = ${}^9C_4 = 126$

65 **(c)**
$$\frac{{}^{56}P_{r+6}}{{}^{54}P_{r+3}} = 30800$$
$$\Rightarrow \frac{56!}{54!} \times \frac{(51-r)!}{(50-r)!} = 30800$$
$$\Rightarrow 56 \times 55 \times (51-r) = 56 \times 55 \times 10$$
$$\Rightarrow 51 - r = 10$$
$$\Rightarrow r = 41$$

66 **(b)**
Obviously, the digit in the middle must be 5. The digits in the first four places must be 1,2,3,4 and the digits in the last four places must be 6,7,8,9.
Hence, required number of numbers = $4! \times 1 \times 4! = 576$

67 **(a)**
In the word INTEGER, we have 5 letters other than 'I' and 'N' of which two are identical (E's). We can arrange these letters in $\frac{5!}{2!}$ ways. In any such arrangements, 'I' and 'N' can be placed in 6 available gaps in ${}^6P_2$ ways.

So, required number of ways $= \frac{5!}{2!} \cdot {}^6P_2 = m_1$.
Now, if word start with 'I' and end with 'R', then the remaining letters are 5

So, total number of ways $= \frac{5!}{2!} = m_2$

$$\therefore \frac{m_1}{m_2} = \frac{5!}{2!} \cdot \frac{6!}{4!} \cdot \frac{2!}{5!} = 30$$

69 **(c)**
Six consonants and three vowels can be selected from 10 consonants and 4 vowels in ${}^{10}C_6 \times {}^4C_3$

ways. Now, these 9 letters can be arranged in 9! Ways.

So, required number of words = $^{10}C_6 \times {}^4C_3 \times 9\,!$

70 **(c)**

We have,

$75600 = 2^4 \cdot 3^3 \cdot 5^2 \cdot 7$

The total number of ways of selecting some or all out of four 2's, three 3's, two 5's and one 7's

$= (4+1)(3+1)(2+1)(1+1) - 1 = 119$

But, this includes the given number itself.

Therefore, the required number of proper factors is 118

71 **(b)**

Required numbers= $5!\left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!}\right]$

=44

72 **(b)**

We have,

$^{35}C_{n+7} = {}^{35}C_{4n-2}$

$\Rightarrow n + 7 + 4n - 2 = 35$ or, $n + 7 = 4n - 2$

$\Rightarrow n = 6$ or, $n = 3$

73 **(b)**

Each man can be given a vote in 3 ways

$\therefore$ Total number of ways $= 3^7$

74 **(b)**

Each question can be omitted or one of the two parts can be attempted i.e. it can be taken in 3 ways.

So, 8 questions can be attempted in $3^8 - 1 =$ 6560 ways

75 **(d)**

Total number of shake hands when each person shake hands with the other once only $= {}^8C_2 = 28$

76 **(a)**

Since, the person is allowed to select at most n coins out of $(2n+1)$ coins, therefore in order to select one, two, three,...., n coins, if T is the total number of ways of selecting at least one coin, then

$T = {}^{2n+1}C_1 + {}^{2n+1}C_2 + \ldots + {}^{2n+1}C_n = 255$ ...(i)

Using the binomial theorem

$^{2n+1}C_0 + {}^{2n+1}C_1 + {}^{2n+1}C_2 + \ldots + {}^{2n+1}C_n$

$+ {}^{2n+1}C_{n+1} + {}^{2n+1}C_{n+2} + \ldots + {}^{2n+1}C_{2n+1}$

$= (1+1)^{2n+1} = 2^{2n+1}$

$\Rightarrow {}^{2n+1}C_0 + 2({}^{2n+1}C_1 + {}^{2n+1}C_2 + \ldots$

$+ {}^{2n+1}C_n) + {}^{2n+1}C_{2n+1} = 2^{2n+1}$

$\Rightarrow 1 + 2(T) + 1 = 2^{2n+1}$

$\Rightarrow 1 + T = \frac{2^{2n+1}}{2} = 2^{2n}$ [from Eq.(i)]

$\Rightarrow 1 + 255 = 2^{2n}$

$\Rightarrow 2^{2n} = 2^8 \Rightarrow n = 4$

77 **(b)**

The number of words that can be formed by the given word is $\frac{9!}{(2!)^3}$

78 **(c)**

Required number

$= {}^6C_1 + {}^6C_2 + {}^6C_3 + {}^6C_4 + {}^6C_5 + {}^6C_6 = 2^6 - 1$

$= 63$

79 **(c)**

In a nine digits number, there are four even places for the four odd digits 3, 3, 5, 5

$\therefore$ Required number of ways $= \frac{4!}{2!2!} \cdot \frac{5!}{2!3!} = 60$

80 **(d)**

A garland can be made form 10 flowers in $\frac{1}{2}(9!)ways$

81 **(b)**

Ten pearls of the one colour can be arranged in $\frac{1}{2}(10-1)\,! = \frac{9!}{2}$ ways

Now, 10 pearls of other colour can be arranged in 10 places between the pearls of first colour in 10! ways

Hence, required number of ways $= \frac{9!}{2} \times 10! = 5 \times (9!)^2$

82 **(c)**

7 women can sit on a round table in $(7-1)\,! = 6\,!$ ways. Now, seven places are created which can be filled by 7 men in $7\,!$ ways

Hence, required number of ways $= 6\,! \times 7\,!$

83 **(a)**

Since each group has 3 persons hence, required number of ways= $\frac{9!}{(3!)^3} = \frac{362880}{6\times6\times6} = 1680$

84 **(a)**

There are 6 letters in the word degree, namely 3 $e's$ and d, g, r. Four letters out of these six can be selected in the following ways :

(i) 3 like letters and 1 different, viz, $eee + d, g$, or r

(ii) 2 like letters and 2 different, viz, ee + any two of d, g, r

(iii) all different letters, viz., $'e\ d\ g\ r'$

So, the total number of ways $= {}^3C_1 + {}^3C_2 + 1 = 7$

85 **(d)**

In the word SACHIN order of alphabets is A, C, H, I, N, S.

The number of words starting with A, C, H, I, N are each equal to 5!

$\therefore$ Total number of wards$5 \times 5! = 600$

The first word starting with S is SACHIN

So, word SACHIN appears at serial number 601

87 **(d)**

Each question can be answered in 4 ways and all question can be answered correctly in only one way.

So, required number of ways $= 4^3 - 1 = 63$

88 **(b)**

Number of friends to be invited=6

Let A, B be the friends who are not to attend the party together. Either none of A, B or one of A, B attend the party

$\therefore$ Number of ways of inviting friends

$= {}^{10-2}C_6 \times {}^2C_0 + {}^{10-2}C_5 \times {}^2C_1$

$= 28 \times 1 + 56 \times 2 = 140$

91 **(b)**

Since, first the 2 women select the chairs amongst 1 to 4 in ${}^4P_2$ ways.

Now, from the remaining 6 chairs three men could be arranged in ${}^6P_3$ ways.

$\therefore$ Total number of arrangements= ${}^4P_2 \times {}^6P_3$

92 **(a)**

Required number of straight lines= ${}^8C_2 - {}^3C_2 + 1 = 26$

93 **(a)**

Since, a five digit number is formed using digits $\{0, 1, 2, 3, 4$ and $5\}$ divisible by 3 *ie*, only possible when sum of digits is multiple of 3 which gives two cases.

Case I {using digits :0, 1, 2, 4, 5}

Number of numbers $= 4 \times 4 \times 3 \times 2 \times 1 = 96$

Case II{using digits 1, 2, 3, 4, 5}

Number of numbers $= 5 \times 4 \times 3 \times 2 \times 1 = 120$

$\therefore$ Total number of formed $120 + 96 = 216$

94 **(c)**

Required number of ways= $9 \times 10 \times 10 \times 10 \times 10$

$= 90000$

95 **(b)**

In a word ARTICLE, there are 7 letters. Out of 7 places, 4 places are odd and 3 even. Therefore 3 vowels can be arranged in 4 odd places in ${}^4P_3$ ways and remaining 4 consonants can be arranged in ${}^4P_4$ways. Hence, required number of ways $= {}^4P_3 \times {}^4P_4 = 576$

96 **(c)**

$\because$ The factors of 9600 $= 2^7 \times 3^1 \times 5^2$

$\therefore$ The number of divisors $= (7 + 1)(1 + 1)(2 + 1)$

$= 8 \times 2 \times 3 = 48$

97 **(d)**

Given four numbers 1, 2, 3 and 4

Number of numbers of one digit= ${}^4P_1 = 4$

Number of numbers of three digit numbers= ${}^4P_3 = 24$

Number of numbers of three digit numbers= ${}^4P_3 = 24$

And four digit numbers= ${}^4P_4 = 24$

$\therefore$ Total number of numbers that can be formed

$= 4 + 12 + 24 + 24 = 64$

98 **(a)**

All strips are of different colours, then the number of flags $= 3! = 6$

When two strips are of same colour, then the number of flags $= {}^3C_1 \frac{3!}{2} . {}^2C_1 = 18$

$\therefore$ Total number of flags= $6 + 18 = 24 = 4!$

99 **(b)**

Factorizing the given number, we have

$38808 = 2^3 \times 3^2 \times 7^2 \times 11$

The total number of divisors of this number is same as the number of ways of selecting some or all of two 2's, two 3's, two 7's and one 11.

Therefore,

The total number of divisors $= (3 + 1)(2 + 1)(1 + 1) - 1$

$= 71$

But, this includes the division by the number itself

Hence, the required number of divisors $= 71 - 1 = 70$

100 **(b)**

Total number of numbers$= 2 \times 2 \times 2 \ldots 10$ times$= 2^{10}$

101 **(b)**

$\because f(x_i) \neq y_i$

ie, no object goes to its scheduled place. Then, number of one-one mappings

$$= 6!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \frac{1}{6!}\right)$$

$$= 6!\left(\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \frac{1}{6!}\right)$$

$$= 360 - 120 + 30 - 6 + 1 = 265$$

102 **(d)**

We have,

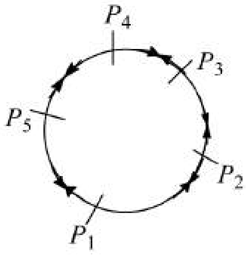
Required number of ways

$$= {}^{m+n}C_m \times (m-1)! \times (n-1)! = \frac{(m+n)!}{m\,n}$$

103 **(a)**

$\because$ Remaining 5 can be seated in 4! ways.

Now, on cross marked five places 2 person can sit in ${}^5P_2$ ways



So, number of arrangements

$$= 4! \times \frac{5!}{3!}$$

$$= 24 \times 20 = 480 \text{ ways}$$

104 **(a)**

Given, ${}^{2n+1}P_{n-1} : {}^{2n-1}P_n = 3:5$

$$\Rightarrow \frac{(2n+1)!}{(n+2)!} \times \frac{(n-1)!}{(2n-1)!} = \frac{3}{5}$$

$$\Rightarrow \frac{(2n+1)2n}{(n+2)(n+1)n} = \frac{3}{5}$$

$$\Rightarrow 10\,(2n+1) = 3(n^2 + 3n + 2)$$

$$\Rightarrow 3n^2 - 11n - 4 = 0$$

$$\Rightarrow (3n+1)(n-4) = 0$$

$$\Rightarrow \; n = 4 \qquad \left(n \neq -\frac{1}{3}\right)$$

105 **(b)**

Required number of ways

$$= {}^4C_1 \times {}^6C_4 + {}^4C_2 \times {}^6C_3 + {}^4C_3 \times {}^6C_2 + {}^4C_4 \times {}^6C_1$$

$$= 60 + 120 + 60 + 6$$

$$= 246$$

106 **(a)**

Required number of ways $= {}^8C_5$

$$= \frac{8 \times 7 \times 6}{3 \times 2 \times 1} = 56$$

The total number of ways a voter can vote

$$= {}^8C_1 + {}^8C_2 + {}^8C_3 + {}^8C_4 + {}^8C_5$$

$$= 8 + 28 + 56 + 70 + 56 = 218$$

107 **(c)**

From the first set, the number of ways of selection two lines $= {}^4C_2$

From the second set, the number of ways of selection two lines $= {}^3C_2$

Since, these sets are intersect, therefore they from a parallelogram,

$\therefore$ Required number of ways $= {}^4C_2 \times {}^3C_2$

$$= 4 \times 3 = 12$$

108 **(b)**

Since, a set of m parallel lines intersecting a set of another n parallel lines in a plane, then the number of parallelograms formed is ${}^mC_2 \times {}^nC_2$.

109 **(a)**

$${}^{50}C_4 + \sum_{r=1}^{6} {}^{56-r}C_3$$

$$= {}^{50}C_4 + {}^{55}C_3 + {}^{54}C_3 + {}^{53}C_3 + {}^{52}C_3 + {}^{51}C_3 + {}^{50}C_3$$

$$= {}^{51}C_4 + {}^{51}C_3 + {}^{52}C_3 + {}^{53}C_3 + {}^{54}C_3 + {}^{55}C_3$$

$[\because {}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1}]$

$$= {}^{52}C_4 + {}^{52}C_3 + {}^{53}C_3 + {}^{54}C_3 + {}^{55}C_3$$

$$= {}^{53}C_4 + {}^{53}C_3 + {}^{54}C_3 + {}^{55}C_3$$

$$= {}^{54}C_4 + {}^{54}C_3 + {}^{55}C_3 = {}^{55}C_4 + {}^{55}C_3 = {}^{56}C_4$$

110 **(a)**

Total number of four digit numbers $= 9 \times 10 \times 10 \times 10$

$= 9000$

Total number of four digit numbers which divisible by 5

$= 9 \times 10 \times 10 \times 2 = 1800$

$\therefore$ Required number of ways $= 9000 - 1800 = 7200$

111 **(a)**

Man goes from Gwalior to Bhopal in 4 ways and they come back in 3 ways.

$\therefore$ Total number of ways$= 4 \times 3 = 12$ ways

112 **(c)**

Here, we have 1 M, 4 I's, 4 S's and 2 P's

$\therefore$ Total number of selections

$= (1+1)(4+1)(2+1) - 1 = 149$

113 **(c)**

Number of lines from 6 points $= {}^6C_2 = 15$

Points of intersection obtained from these lines $= {}^{15}C_2 = 105$

Now, we find the number of times, the original 6 points come.

Consider one point say A_1.Joining A_1 to remaining 5 points, we get 5 lines and any two lines from these 5 lines gives A_1 as the point of intersection.

$\therefore A_1$ is commom in ${}^5C_2 = 10$ times out of 105 points of intersections.

Similar is the case with other five points.

$\therefore$ 6 original points come $6\times 10 = 60$ times in points of intersection.

Hence, the number of distinct points of intersection

$= 105 - 60 + 6 = 51$

115 **(b)**

At first we have to a accommodate those 5 animals in cages which cannot enter in 4 small cages, therefore, number of ways are ${}^6P_5$ and rest of the five animals arrange in 5! ways.

Total number of ways $= 5! \times {}^6P_5$

$= 120 \times 720 = 86400$

116 **(b)**

$T_n = {}^nC_3$ and $T_{n+1} - T_n = 21$

$\Rightarrow {}^{n+1}C_3 - {}^nC_3 = 21$

$\Rightarrow {}^nC_2 + {}^nC_3 - {}^nC_3 = 21$

$\Rightarrow {}^nC_2 = 21$

$\Rightarrow \dfrac{n(n-1)}{2} = 21$

$\Rightarrow n^2 - n - 42 = 0$

$\Rightarrow (n-7)(n+6) = 0$

$\therefore n = 7 \qquad [\because \neq -6]$

117 **(b)**

Total number of ways

$= {}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + {}^{10}C_4$

$= 10 + 45 + 120 + 210 = 385$

118 **(b)**

The total number of two factors product$= {}^{n+2}C_8$.

The number of numbers from 1 to 200 which are not multiples of 5 is 160. Therefore, total number of two factors product, which are not multiple of 5, is ${}^{160}C_2$

Hence, required number of factors$= {}^{200}C_2 - {}^{160}C_2$

$= 19900 - 12720$

$= 7180$

119 **(b)**

Total number of m-elements subsetcs of $A = {}^nC_m$ ...(i)

and number of m-elements subsets of A each containing the element $a_4 = {}^{n-1}C_{m-1}$

According to question, ${}^nC_m = \lambda.\, {}^{n-1}C_{m-1}$

$\Rightarrow \dfrac{n}{m}.{}^{n-1}C_{m-1} = \lambda.{}^{n-1}C_{m-1}$

$\Rightarrow \lambda = \dfrac{n}{m}$ or $n = m\lambda$

120 **(a)**

The number of 1 digit numbers= 9

The number of 2 digit non-repeated numbers= $9 \times 9 = 81$

The number of 3 digit non-repeated number

$= 9 \times {}^9P_2 = 9 \times 9 \times 8 = 648$

$\therefore$ Required number of ways $=9+81+648=738$

121 **(b)**

Now, ${}^nC_{r+1} + {}^nC_{r-1} + 2.\ {}^nC_r$

$= {}^nC_{r+1} + {}^nC_r + {}^nC_{r-1} + {}^nC_r$

$= {}^{n+1}C_{r+1} + {}^{n+1}C_r = {}^{n+2}C_{r+1}$

122 **(a)**

$\frac{2}{9!} + \frac{2}{3!\,7!} + \frac{1}{5!\,5!}$

$= \frac{1}{1!\,9!} + \frac{1}{3!\,7!} + \frac{1}{5!\,5!} + \frac{1}{3!\,7!} + \frac{1}{9!\,1!}$

$= \frac{1}{10!}\left[\frac{10!}{1!\,9!} + \frac{10!}{3!\,7!} + \frac{10!}{5!\,5!} + \frac{10!}{3!\,7!} + \frac{10!}{9!\,1!}\right]$

$= \frac{1}{10!}\{{}^{10}C_1 + {}^{10}C_3 + {}^{10}C_5 + {}^{10}C_7 + {}^{10}C_9\}$

$= \frac{1}{10!}(2^{10-1}) = \frac{2^9}{10!} = \frac{2^a}{b!}$ (given)

$\Rightarrow a = 9, b = 10$

123 **(c)**

Total number of lines obtained by joining 8 vertices of octagon is ${}^8C_2 = 28$. Out of these, 8 lines are sides and remaining diagonal.

So, number of diagonals $= 28 - 8 = 20$

124 **(b)**

The number of times he will go to the garden is same as the number of selecting 3 children from 8 children

$\therefore$ The required number of times= ${}^8C_3 = 56$

125 **(c)**

$\because \quad {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

$\therefore \quad {}^{189}C_{36} + {}^{189}C_{35} = {}^{190}C_{36}$

But ${}^{189}C_{35} + {}^{189}C_x = {}^{190}C_x$

Hence, value of x is 36

126 **(d)**

Required number of ways = ${}^{3n}C_n = \frac{3n!}{n!\,2n!}$

127 **(a)**

The word EXAMINATION has 2A, 2I, 2N, E, M, O, T, X therefore 4 letters can be chosen in following ways

Case I When 2 alike of one kind and 2 alike of second kind is ${}^3C_2$

$\therefore$ Number of words= ${}^3C_2 \times \frac{4!}{2!2!} = 18$

Case II When 2 alike of one kind and 2 different *ie*, ${}^3C_1 \times {}^7C_2$

$\therefore$ Number of words= ${}^3C_1 \times {}^7C_2 \times \frac{4!}{2!} = 756$

Case III When all are different *ie*, ${}^8C_4$

Hence, total number of words

$= 18 + 756 + 1680 = 2454$

128 **(a)**

Required number of ways= $5! \times 6!$

129 **(d)**

Number of diagonals in a polygon of n sides

$= {}^nC_2 - n$

Here, $n = 20$

$\therefore$ required number of diagonals = ${}^{20}C_2 - 20$

$= \frac{20 \times 19}{2 \times 1} - 20 = 170$

130 **(c)**

${}^{47}C_4 + \sum_{r=1}^{5} {}^{52-r}C_3$

$= {}^{47}C_4 + {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3$

$= {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + ({}^{47}C_3 + {}^{47}C_4)$

$= {}^{52}C_4$

131 **(a)**

First we fix the alternate position of 21 English book, in which 22 vacant places for Hindi books, hence total number of ways are ${}^{22}C_{19} = 1540$

132 **(d)**

Required number of ways

= Total number of ways in which 8 boys can sit

− Number of ways in which two brothers sit together

$= 8! - 7! \times 2! = 7! \times 6 = 30240$

133 **(c)**

In forming even numbers, the position on the right can be filled with either 0 or 2. When 0 is filled, the remaining positions can be filled in 3! ways, and when 2 is filled, the position on the left can be filled in 2 ways (0 cannot be used) and the middle two positions in 2! ways (0 can be used) So, the number of even numbers formed = $3! + 2(2!) = 0$

135 **(a)**

Let the number of participants at the beginning was n

$\therefore \quad \frac{n(n-1)}{2} = 117 - 12$

$\Rightarrow \quad n(n-1) = 2 \times 105$

$\Rightarrow \quad n^2 - n - 210 = 0$

$\Rightarrow \quad (n-15)(n+14) = 0$

$\Rightarrow \quad n = 15 \quad [\because n \neq -14]$

136 **(a)**

The number will be even if last digit is either 2, 4, 6 or 8 *ie* the last digit can be filled in 4 ways and remaining two digits can be filled in ${}^{8}P_2$ ways. Hence, required number of number of three different digits = ${}^{8}P_2 \times 4 = 224$

137 **(b)**

We have, $a = {}^{x+2}P_{x+2} = (x+2)!$,

and $b = {}^{x}P_{11} = \frac{x!}{(x-11)!}$

and $c = {}^{x-11}P_{x-11} = (x-11)!$

Now, $a = 182\ bc$

$\therefore\ (x+2)! = 182.\dfrac{x!}{(x-11)!}(x-11)!$

$\Rightarrow (x+2)! = 182\ x!$

$\Rightarrow (x+2)(x+1) = 182$

$\Rightarrow x^2 + 3x - 180 = 0$

$\Rightarrow (x-12)(x+15) = 0$

$\Rightarrow x = 12, -15$

$\therefore$ Neglect the negative value of x.

$\Rightarrow\ x = 12$

138 **(c)**

Since, the books consisting of 5 Mathematics, 4 physics, and 2 chemistry can be put together of the same subject is 5! 4! 2! ways

But these subject books can be arranged itself in 3! ways

$\therefore$ Required number of ways = 5! 4! 3! 2!

139 **(a)**

If the function is one-one, then select any three from the set B in ${}^{7}C_3$ ways $i.e.$, 35 ways.

If the function is many-one, then there are two possibilities. All three corresponds to same element number of such functions = ${}^{7}C_1 = 7$ ways. Two corresponds to same element. Select any two from the set B. The lerger one corresponds to the larger and the smaller one corresponds to the smaller the third may corresponds to any two. Number of such functions = ${}^{7}C_2 \times 2 = 42$

So, the required number of mappings = 35 + 7 + 42 = 84

140 **(b)**

The number of ordered triples of positive integers which are solution of $x + y + z = 100$

=coefficient of x^{100} in $(x + x^2 + x^3 + ..)^3$

=coefficient of x^{100}in $x^3(1-x)^{-3}$

=coefficient of x^{97}in

$$\left(1 + 3x + 6x^2 + \dots\dots + \frac{(n+1)(n+2)}{2}x^n + \dots\right)$$

$$= \frac{(97+1)(97+2)}{2} = 49 \times 99 = 4851$$

141 **(b)**

Word MEDITERRANEAN has 2A, 3E, 1D, 1I, 1M, 2N, 2R, 1T

In out of four letters E and R is fixed and rest of the two letters can be chosen in following ways

Case I Both letter are of same kind *ie*, ${}^{3}C_2$ ways, therefore number of words = ${}^{3}C_2 \times \frac{2!}{2!} = 3$

Case II Both letters are of different kinds *ie*, ${}^{8}C_2$ ways, therefore number of words= ${}^{8}C_2 \times 2! = 56$

Hence, total number of words=56+3=59

142 **(c)**

Required number of ways

=coefficient of x^{2m}in $(x^0 + x^1 + \dots + x^m)^4$

=coefficient of x^{2m} in $\left(\frac{1-x^{m+1}}{1-x}\right)^4$

=coefficient of x^{2m} in $(1 - 4x^{m+1} + 6x^{2m+2} + \dots)(1-x)^{-4}$

$= {}^{2m+3}C_{2m} - 4\,{}^{m+2}C_{m-1}$

$$= \frac{(2m+1)(2m+2)(2m+3)}{6} - \frac{4m(m+1)(m+2)}{6}$$

$$= \frac{(m+1)(2m^2+4m+3)}{3}$$

143 **(c)**

The number of times he will go to the garden is same as the number of selecting 3 children from 8.

Therefore, the required number of ways $= {}^8C_3 = 56$

144 **(c)**

The number of ways that the candidate may select

(i) if 2 questions from A and 4 question from B

$= {}^5C_2 \times {}^5C_4 = 50$

(ii) 3 question from A and 3 questions from B

$= {}^5C_3 \times {}^5C_3 = 100$

and (iii) 4 questions from A and 2 questions from B

$= {}^5C_4 \times {}^5C_2 = 50$

Hence, total number of ways $= 50 + 100 + 50 = 200$

145 **(a)**

Since, $240 = 2^4.3.5$

$\therefore$ Total number of divisors $= (4+1)(1+1)(1+1) = 20$

Out of these 2, 6, 10 and 30 are of the form $4n + 2$

147 **(a)**

Required number of arrangements

$= \frac{6!}{2!\,3!} - \frac{5!}{3!} = 60 - 20 = 40$

148 **(b)**

As we know the last two digits of 10! and above factorials will be zero-zero

$\therefore \quad 1! + 4! + 7! + 10! + 12! + 13! + 15! + 16! + 17!$

$= 1 + 24 + 5040 + 10! + 12! + 13! + 15! + 16! + 17!$

$= 5065 + 10! + 12! + 13! + 15! + 16! + 17!$in this series, the digit in the ten palce is 6 which is divisible by 3!

149 **(c)**

As the players who are to receive the cards are different

So, the required number of ways $= \frac{52\,!}{(13\,!)^4}$

150 **(c)**

We have, in all 12 points. Since 3 points are used to form a triangle, therefore the total number of triangles, including the triangles formed by collinear points on AB, BC and CA, is ${}^{12}C_3 = 220$.

But, this includes the following:

The number of triangles formed by 3 points on AB $= {}^3C_3 = 1$,

The number of triangles formed by 4 points on BC $= {}^4C_3 = 4$,

The number of triangles formed by 5 points on CA $= {}^5C_3 = 10$,

Hence, required number of triangles $= 220 - (10 + 4 + 1) = 205$

151 **(b)**

Given, $\quad {}^nP_r = 3024$

$\Rightarrow \quad \frac{n!}{(n-r)!} = 3024$

And $\quad {}^nC_r = \frac{n!}{r!(n-r)!}$

$\Rightarrow \quad 126 = \frac{3024}{r!}$

$\Rightarrow \quad r! = 24 = 4!$

$\Rightarrow \quad r = 4$

152 **(d)**

We have,

$${}^{35}C_8 + \sum_{r=1}^{7} {}^{42-r}C_7 + \sum_{s=1}^{5} {}^{47-s}C_{40-s}$$

$= {}^{35}C_8 + \{{}^{41}C_7 + {}^{40}C_7 + {}^{39}C_7 + {}^{38}C_7 + \cdots + {}^{35}C_7\}$

$+\{{}^{46}C_{39} + {}^{45}C_{38} + \cdots + {}^{42}C_{35}\}$

$= {}^{35}C_8 + \{{}^{35}C_7 + {}^{36}C_7 + \cdots + {}^{41}C_7\}$

$+\{{}^{42}C_7 + {}^{43}C_7 + \cdots + {}^{46}C_7\} \qquad [\because {}^nC_r = {}^nC_{n-r}]$

$= ({}^{35}C_8 + {}^{35}C_7) + ({}^{36}C_7 + \cdots + {}^{41}C_7 + \cdots + {}^{46}C_7)$

$= ({}^{36}C_8 + {}^{36}C_7) + {}^{37}C_7 + \cdots + {}^{46}C_7$

$= {}^{37}C_8 + {}^{37}C_7 + {}^{38}C_7 + \cdots + {}^{46}C_7$

$= \ldots\ldots\ldots\ldots\ldots$

$= {}^{46}C_8 + {}^{46}C_7 = {}^{47}C_8$

153 **(b)**

Taking A_1, A_2 as one group we have 9 candidates which can be ranked in 9! ways. But A_1 and A_2 can be arranged among themselves in 2 ! ways

Hence, the required number $= (9\,!)(2\,!) = 2(9\,!)$

154 **(c)**

Considering $A\mathcal{U}$ as one letter, we have 4 letters, namely $L, A\mathcal{U}, G, H$ which can be permuted in 4! ways. But, A and $\mathcal{U}$ can be put together in 2! Ways.

Thus, the required number of arrangements $= 4\,! \times 2\,! = 48$

155 **(c)**

Total number of ways in which all letters can be arranged in 6! ways.

There are two vowels in the word GARDEN

Total number of ways in which these two vowels can be arranged= 2!

∴ Total number of required ways= $\frac{6!}{2!} = 360$

156 **(a)**

The possible cases are

Case I A man invites 3 ladies and woman invites 3 gentleman

$\Rightarrow \quad {}^4C_3\, {}^4C_3 = 16$

Case II A man invites (2 ladies, 1 gentlemen) and woman invites (2 gentlemen, 1 lady)

$\Rightarrow \quad ({}^4C_2 . {}^3C_1).({}^3C_1 . {}^4C_2) = 324$

Case III A man invites (1 lady, 2 gentlemen) and woman invites (2 ladies, 1 gentlemen)

$\Rightarrow \quad ({}^4C_1 . {}^3C_2).({}^3C_2 . {}^4C_1) = 144$

Case IV A man invites (3 gentlemen) and woman invites (3 ladies)

$\Rightarrow \quad {}^3C_3 . {}^3C_3 = 1$

∴ Total number of ways

$= 16 + 324 + 144 + 1 = 485$

158 **(a)**

A number between 5000 and 10,000 can have any of the digits 5,6,7,8,9 at thousand's place. So, thousand's place can be filled in 5 ways.

Remaining 3 places can be filled by the remaining 8 digits in ${}^8P_3$ ways

Hence, required number $= 5 \times {}^8P_3$

160 **(d)**

Two circles intersect maximum at two distinct points. Now, two circles can be selected in ${}^6C_2$ ways.

∴ Total number of points in intersection are

${}^6C_2 \times 2 = 30$

161 **(b)**

The numbers formed will be divisible by 4 if the number formed by the two digits on the extreme right is divisible by 4, i.e. it should be 04,12,20,24,32,40

The number of numbers ending in $04 = 3\,! = 6$

The number of numbers ending in $12 = 3\,! - 2\,! = 4$

The number of numbers ending in $20 = 3\,! = 6$

The number of numbers ending in $24 = 3\,! - 2\,! = 4$

The number of numbers ending in $32 = 3\,! - 2\,! = 4$

The number of numbers ending in $40 = 3\,! = 6$

So, the required number $= 6 + 4 + 6 + 4 + 4 + 6 = 30$

162 **(c)**

The four girls can first be arranged in 4 ! ways among themselves. In each of these arrangements there are 5 gaps (including the extremes) among the girls. Since the boys and girls are to alternate, we have to leave the first gap or last gap blank while arranging the boys. But, in each case the boys and girls can be arranged in $4\,! \cdot 4\,!$ ways

∴ Required number of ways $= 2(4\,! \times 4\,!) = 2(4\,!)^2$

163 **(a)**

The product of r consecutive natural numbers

$= 1.2.3.4 \ldots\ldots r = r!$

The natural number will divided by $r!$

164 **(d)**

The number of ways in which at least 5 women can be included in a committee

$$= {}^9C_5 \times {}^8C_7 + {}^9C_6 \times {}^8C_6 + {}^9C_7 \times {}^8C_5 + {}^9C_8 \times {}^8C_4 + {}^9C_9 \times {}^8C_3$$

(i) Women are in majority, then number of ways

$= {}^9C_7 \times {}^8C_5 + {}^9C_8 \times {}^8C_4 + {}^9C_9 \times {}^8C_3$

$= 2016 + 630 + 56 = 2702$

(ii) Men are in majority, then number of ways

$= {}^9C_5 \times {}^8C_7 = 126 \times 8 = 1008$

165 **(c)**

In the number which is divisible by 5 and lying between 3000 and 4000, 3 must be at thousand place and 5 must be at unit place. Therefore rest of the digits (1, 2, 3, 4, 6) fill in two places. The number of ways= ${}^4P_2$

166 **(b)**

We have 12 letters including 2 C's. Let us ignore 2 C's and thus we have 10 letters $(4\, A's, 3\, B's, 1\, D, 1\, E, 1\, F)$ and these 10 letters can be arrange in $\frac{10\,!}{4\,!3\,!}$ ways.

Now, after arranging these 10 letters there will be 11 gaps in which two different letters can be arranged in ${}^{11}P_2$ ways. But, since 2 C's are alike, the number of arrangements will be $\frac{1}{2!}\ {}^{11}P_2 = \frac{11!}{9!2!}$

So, total number of ways in which C's are separated from one another $= \frac{10!}{4!3!} \cdot \frac{11!}{9!2!} =$ 1386000

167 **(d)**

An odd number has an odd digit at unit's place

So, unit's place can be filled in 4 ways

Each of ten's and hundred's place can be filled in 6 ways

Thousand place can be filled in 5 ways

Hence, required number of numbers$= 5 \times 6 \times 6 \times 4 = 720$

168 **(c)**

$\because\ {}^{12}P_r = 1320 = 12 \times 11 \times 10$

$\Rightarrow \dfrac{12!}{(12-r)!} = 12 \times 11 \times 10$

$\therefore\ r = 3$

169 **(c)**

Total time required=(total number of dials required to sure open the lock)$\times 5s$

$= 10^5 \times 5s$

$= \dfrac{500000}{60 \times 60 \times 13}$ days $= 10.7$ days

Hence, 11 days are enough to open the safe.

170 **(a)**

There are 6 rings and 4 fingers.

Since, each ring can be worn on any finger.

$\therefore$ Required number of ways$= 4^6$

171 **(c)**

Consider the product

$(x^0 + x^1 + x^2 \dots + x^9)(x^0 + x^1 + x^2 \dots + x^6) \dots 6$ factors

The number of ways in which the sum of the digits will be equal to 12 is equal to the coefficient of x^{12} in the above product. So, required number of ways

$=$ Coeff. Of x^{12} in $\left(\frac{1-x^{10}}{1-x}\right)^6$

$=$ Coeff. Of x^{12} in $(1-x^{10})^6(1-x)^{-6}$

$=$ Coeff. Of x^{12} in $(1-x)^{-6}(1 - {}^6C_1 x^{10} + \cdots)$

$=$ Coeff. Of x^{12} in $(1-x)^{-6} - {}^6C_1 \cdot$ Coeff. of x^2 in $(1-x)^{-6}$

$= {}^{12+6-1}C_{6-1} - {}^6C_1 \times {}^{2+6-1}C_{6-1}$

$= {}^{17}C_5 - 6 \times {}^7C_5 = 6062$

172 **(c)**

We observe that a point is obtained between the lines of two of points on first line are joined by line segments to two points on the second line

Hence, required number of points $= {}^nC_2 \times {}^nC_2$

174 **(c)**

Let there be 'n' men participants. Then, the number of games that the men play between themselves is 2. ${}^nC_2$ and the number of games that the men played with the women is 2. $(2n)$

$\therefore$ 2. ${}^nC_2 - 2.2n = 66$ (given)

$\Rightarrow n(n-1) - 4n - 66 = 0$

$\Rightarrow n^2 - 5n - 66 = 0$

$\Rightarrow (n+5)(n-11) = 0$

$\Rightarrow n = 11$

$\therefore$ Number of participants =11 men+2 women=13

175 **(b)**

There are total $20 + 1 = 21$ persons. The two particular persons and the host be taken as one unit so that these remaining $21 - 3 + 1 = 19$ persons be arranged in round table in 18! ways.

But the two persons on either side of the host can themselves be arranged in 2! ways

$\therefore$ required number of ways$= 2! \times 18!$

176 **(b)**

Let the total number of persons in the room $= n$

$\therefore$ Total number of handshakes$= {}^nC_2 = 66$ (given)

$\Rightarrow \frac{n!}{2!(n-2)!} = 66 \ \Rightarrow \frac{n(n-1)}{2} = 66$

$\Rightarrow\ n^2 - n - 132 = 0$

$\Rightarrow\ (n-12)(n+11) = 0$

$\Rightarrow\ n = 12 \quad [\because n \neq -11]$

177 **(d)**

Given word is 'PENCIL'.

Total alphabets in the given word=6

Number of vowels=2 and number of consonants=4

$\because$4 consonants can be arranged in 4! ways.

$\therefore$ Remaining two places can be filled by two vowels in ${}^5P_2$ ways.

$\therefore$ Total number of ways$4! \times {}^5P_2 = 24 \times 20 = 480$

180 **(b)**

Let there are n teams.

Each team play to every other team in ${}^{n}C_{23}$ ways

$\therefore {}^{n}C_2 = 153$ (given)

$\Rightarrow \dfrac{n!}{(n-2)!\,2!} = 153$

$\Rightarrow n(n-1) = 306$

$\Rightarrow n^2 - n - 306 = 0$

$\Rightarrow (n-18)(n+17) = 0$

$\Rightarrow n = 18 \quad (\because n \text{ is never negative})$

181 **(a)**

Since total number are 15, but three special members constitute one member.

Therefore, required number of arrangements are $12! \times 2$, because, chairman remains between the two specified persons and person can sit in two ways

182 **(b)**

Let there be n participants. Then, we have

${}^{n}C_2 = 45$

$\Rightarrow \dfrac{n(n-1)}{2} = 45 \Rightarrow n^2 - n - 9 = 0 \Rightarrow n = 10$

183 **(d)**

Required number of ways $= {}^{12-1}C_{9-1}$

$= {}^{11}C_8 = \dfrac{11 \times 10 \times 9}{3 \times 2 \times 1} = 165$

185 **(c)**

A number is divisible by 3, if the sum of the digits is divisible by 3

Since, 1+2+3+4+5=15 is divisible by 3, therefore total such numbers is 5! *ie*, 120

And, other five digits whose sum is divisible by 3 are 0, 1, 2, 4, 5

Therefore, number of such formed numbers= $5! - 4! = 96$

Hence, the required number if numbers=120+96=216

187 **(b)**

Each child will go as often as he (or she) can be accompanied by two others

$\therefore$ Required number $= {}^{7}C_2 = 21$

188 **(a)**

We have,

Required sum $= (2+3+4+5)(4-1)!\left(\frac{10^4-1}{10-1}\right)$

$= 14 \times 6 \times \left(\dfrac{10^4 - 1}{10 - 1}\right) = 93324$

189 **(b)**

Here, we have to divide 52 cards into 4 sets, three of them having 17 cards each and the fourth one having just one card. First we divide 52 cards into two groups of 1 card and 51 cards. This can be done in $\frac{52!}{1!\,51!}$ ways

Now, every group of 51 cards can be divided into 3 groups of 17 each in $\frac{51!}{(17!)^3\,3!}$

Hence, the required number of ways

$= \dfrac{52!}{1!\,51!} \cdot \dfrac{51!}{(17!)^3 3!} = \dfrac{52!}{(17!)^3 3!}$

190 **(d)**

The required number is ${}^{9}C_5 + {}^{9}C_4 \times {}^{8}C_1 + {}^{9}C_3 \times {}^{8}C_2 = 3486$

191 **(c)**

If there were no three points collinear, we should have ${}^{10}C_2$ lines; but since 7 points are collinear we must subtract ${}^{7}C_2$ lines and add the one corresponding to the line of collinearity of the seven points.

Thus, the required number of straight lines = ${}^{10}C_2 - {}^{7}C_2 + 1 = 25$

193 **(d)**

The required number of points

$= {}^{8}C_2 \times 1 + {}^{4}C_2 \times 2 + ({}^{8}C_1 \times {}^{4}C_1) \times 2$

$= 28 + 12 + 32 \times 2 = 104$

194 **(d)**

${}^{16}C_r = {}^{16}C_{r+1}$

$\Rightarrow \quad {}^{16}C_{16-r} = {}^{16}C_{r+1} \quad [\because {}^{n}C_r = {}^{n}C_{n-r}]$

$\Rightarrow \; 16 - r = r + 1 \;\Rightarrow\; 2r = 15$

$\Rightarrow \quad r = 7.5$

Which is not possible, since r should be an integer

195 **(a)**

We have,

$$\sum_{r=0}^{m} {}^{n+r}C_n = \sum_{r=0}^{m} {}^{n+r}C_r \qquad [\because {}^{n}C_r = {}^{n}C_{n-r}]$$

$$\Rightarrow \sum_{r=0}^{m} {}^{n+r}C_n = {}^{n}C_0 + {}^{n+1}C_1 + {}^{n+2}C_2 + \cdots + {}^{n+m}C_m$$

$$\Rightarrow \sum_{r=0}^{m} {}^{n+r}C_n = [1 + (n+1)] + {}^{n+2}C_2 + {}^{n+3}C_3 + \cdots + {}^{n+m}C_m$$

$$\Rightarrow \sum_{r=0}^{m} {}^{n+r}C_n = ({}^{n+2}C_1 + {}^{n+2}C_2) + {}^{n+3}C_3 + \cdots + {}^{n+m}C_m$$

$$\Rightarrow \sum_{r=0}^{m} {}^{n+r}C_n = ({}^{n+3}C_2 + {}^{n+3}C_3) + {}^{n+4}C_4 + \cdots + {}^{n+m}C_m$$

$$\Rightarrow \sum_{r=0}^{m} {}^{n+r}C_n = ({}^{n+4}C_3 + {}^{n+4}C_4) + \cdots + {}^{n+m}C_m$$

... ...

$$= {}^{n+m}C_{m-1} + {}^{n+m}C_m$$

$$\Rightarrow \sum_{r=0}^{m} {}^{n+r}C_n = {}^{n+m+1}C_m$$

$$\Rightarrow \sum_{r=0}^{m} {}^{n+r}C_n = {}^{n+m+1}C_{n+1} \quad [\because {}^nC_r = {}^nC_{n-r}]$$

196 **(a)**

Taking option (a)

$${}^{n-1}P_r + r\,{}^{n-1}P_{r-1} = \frac{(n-1)!}{(n-1-r)!} + \frac{(n-1)!}{(n-r)!}$$

$$\left(\therefore {}^nP_r = \frac{n!}{(n-r)!}\right)$$

$$= \frac{(n-1)!}{(n-1-r)!}\left(1 + r.\frac{1}{n-r}\right)$$

$$= \frac{(n-1)!}{(n-1-r)!}\left(\frac{n}{n-r}\right) = \frac{n!}{(n-r)!} = {}^nP_r$$

197 **(a)**

First we fix the position of 6 men, the number of ways to sit men= 5! and the number of ways to sit women ${}^6P_5$

$\therefore$ Total number of ways = 5! ${}^6P_5$ = 5! × 6!

198 **(b)**

In a octagon there are eight sides and eight points

$\therefore$ Required number of diagonals

$= {}^8C_2 - 8 = 28 - 8 = 20$

199 **(a)**

The required number of ways=The even number of 0's *ie*, {0, 2, 4, 6, ...}

$$= \frac{n!}{n!} + \frac{n!}{2!\,(n-2)!} + \frac{n!}{4!\,(n-4)!} + \cdots$$

$$= {}^nC_0 + {}^nC_2 + {}^nC_4 + \ldots = 2^{n-1}$$

200 **(c)**

We have,

$${}^{n-1}C_3 + {}^{n-1}C_4 > {}^nC_3$$

$$\Rightarrow {}^nC_4 > {}^nC_3 \quad [\because {}^nC_{r-1} + {}^nC_r = {}^{n+1}C_r]$$

$$\Rightarrow \frac{n\,!}{(n-4)\,!\,4\,!} > \frac{n\,!}{(n-3)\,!\,3\,!}$$

$$\Rightarrow \frac{1}{4} > \frac{1}{n-3} \Rightarrow n > 7$$

201 **(d)**

Let the total number of contestants= n

A voter can vote to $(n-1)$ candidates

$\therefore\ {}^nC_1 + \cdots + {}^nC_{n-1} = 126$

$\Rightarrow\ 2^n - 2 = 126$

$\Rightarrow\ 2^n = 128 = 2^7$

$\Rightarrow\ n = 7$

202 **(c)**

We have,

$${}^nC_{n-r} + 3\cdot{}^nC_{n-r+1} + 3\cdot{}^nC_{n-r+2} + {}^nC_{n-r+3} = {}^xC_r$$

$$\Rightarrow ({}^nC_{n-r} + {}^nC_{n-r+1}) + 2({}^nC_{n-r+1} + {}^nC_{n-r+2}) + ({}^nC_{n-r+2} + {}^nC_{n-r+3}) = {}^xC_r$$

$$\Rightarrow {}^{n+1}C_{n-r+1} + 2\,{}^{n+1}C_{n-r+2} + {}^{n+1}C_{n-r+3} = {}^rC_r$$

$[\because {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r]$

$$\Rightarrow \{{}^{n+1}C_{n-r+1} + {}^{n+1}C_{n-r+2}\} + \{{}^{n+1}C_{n-r+2} + {}^{n+1}C_{n-r+3}\} = {}^xC_r$$

$$\Rightarrow {}^{n+2}C_{n-r+2} + {}^{n+2}C_{n-r+3} = {}^xC_r$$

$$\Rightarrow {}^{n+3}C_{n-r+3} = {}^xC_r$$

$$\Rightarrow {}^{n+3}C_r = {}^xC_r \quad [\because {}^{n+3}C_{n-r+3} = {}^{n+3}C_r]$$

$$\Rightarrow x = n + 3$$

203 **(b)**

A 2 × 2 matrix has 4 elements such that each element can two values. Thus, total number of matrices

= 2 × 2 × 2 × 2 = 16

204 **(a)**

$\because$ Total number of seats= n

and number of people= m

Ist person can be seated in n ways

IInd person can be seated in $(n-1)$ ways

... ...

... ...

... ...

mth person can be seated in $(n-m+1)$*ways*

$\therefore$ Total number of ways

$= n(n-1)(n-2)\ldots(n-m+1) = {}^nP_m$

Alternate In out of n seats m people can be seated in ${}^nP_m$ ways

205 **(c)**

Given word is EAMCET

Here number of vowels are 3 *ie*, E, A, E and number of consonants are 3, *ie*, M, C, T and number of ways of arranging three consonants= 3! = 6

V C V C V C V

In the places 'V', we shall arrange vowels

There are 4 places marked V

∴ Number of ways of arranging vowels

$= {}^4P_3 + \frac{1}{2} = 12$ [∵ E is repeated twice]

∴ Total number of words= $6 \times 12 = 72$

207 **(d)**

The vowels in the word "COMBINE" are O, I, E which can be arranged at 4 places in ${}^4P_3$ ways and other words can be arranged in 4! ways

Hence, total number of ways= ${}^4P_3 \times 4!$

$= 4! \times 4!$

$= 576$

208 **(b)**

Number of ways of giving one prize for running = 16

Number of ways of giving one prizes for swimming $= 16 \times 15$

Number of ways of giving three prizes for riding = $16 \times 15 \times 14$

∴ Required ways of giving prizes = $16 \times 16 \times 15 \times 16 \times 15 \times 14$

$= 16^3 \times 15^2 \times 14$

209 **(b)**

First we fix the alternate position of girls and they arrange in 4! ways and in the five places five boys can be arranged in ${}^5P_5$ ways

∴ Total number of ways= $4! \times {}^5P_5 = 4! \times 5!$

210 **(c)**

Number of vertices=15

∴ Number of lines=${}^{15}C_2 = 105$

∴ Number of diagonals=105-15=90

211 **(b)**

At least one green ball can be selected out of 5 green balls in $2^5 - 1 = 31$ ways. Similarly at least one blue ball can be selected from 4 blue balls in $2^4 - 1 = 15$ ways and at least one red or not red can be select in $2^3 = 8$ ways

Hence ,required number of ways = $31 \times 15 \times 8 =$ 3720

212 **(c)**

We have,

The required number of words

$= ({}^2C_1 \times {}^4C_2 + {}^2C_2 \times {}^4C_1)\, 3\,! = 96$

213 **(c)**

First deduct the n things and arrange the m things in a row taken all at a time, which can be done in m! ways. Now in $(m+1)$ spaces between them (including the beginning and end) put the n things one in each space in all possible ways. This can be done in ${}^{m+1}P_n$ ways.

So, the required number $= m\,!\ {}^{m+1}P_n = \frac{m\,!\,(m+1)\,!}{(m+1-n)\,!}$

214 **(b)**

Number greater than 1000 and less than or equal to 4000 will be of 4 digits and will have either 1 (expect 1000) or 2 or 3 in the first place with 0 in each of remaining places.

After fixing 1st place, the second place can be filled by any of the 5 numbers.

Similarly, third place can be filled up in 5 ways and 4th place can be filled up in 5 ways. Thus, there will be $25 \times 5 = 125$

Ways in which 1 will be in first place but this include 1000 also hence there will be 124 numbers having 1 in the first place. Similarly, 125 ways for each 2 or 3 in the Ist place.

One number will be in which 4 in the first place *ie*, 4000

Hence, the required number of numbers

$= 124 + 125 + 125 + 1 = 375$

215 **(a)**

Considering four particular flowers as one flower, we have five flowers which can be strung to form a garland in 4! ways. But, 4 particular flowers can be arranged in 4! ways. Thus, the required number $= 4\,! \times 4\,!$

216 **(b)**

Any number between 1 to 999 is a 3 digit number xyz where the digits x, y, z are any digits from 0 to 9.

Now, we first count the number in which 3 occurs once only. Since 3 can occur at one place in ${}^3C_1$ways, there are ${}^3C_1.(9 \times 9) = 3.9^2$ such numbers.

Again, 3 occur in exactly two places in ${}^3C_2.(9)$ such numbers. Lastly 3 can occur in all the three digits in one such number only 333

∴ The number of times 3 occurs

$= 1 \times (3 \times 9^2) + 2 \times (3 \times 9) + 3 \times 1 = 300$

217 **(b)**

Number of triangles= ${}^{n+3}C_3 = 220$

$\Rightarrow \quad \frac{(n+3)!}{3!\,n!} = 220$

$\Rightarrow \quad (n+1)(n+2)(n+3) = 1320$

$= 12 \times 10 \times 11$

$= (9+1)(9+2)(9+3)$

$\therefore \quad n = 9$

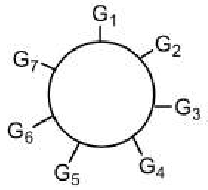
218 **(a)**

First we fix the alternate position of 7 gentlemen in a round table by 6! ways.

There are seven positions between the gentlemen in which 5 ladies can be seated in ${}^7P_5$ ways

$\therefore$ required number of ways

$= 6! \times \frac{7!}{2!} = \frac{7}{2}(720)^2$



219 **(c)**

The number between 999 and 10000 are of four digit numbers.

The number of four digit numbers formed by digits 0, 2, 3, 6, 7, 8 is ${}^6P_4 = 360$

But here those numbers are also involved which being from 0.

So, we take those numbers as three digit numbers.

Taking initial digit 0, the number of ways to fill remaining 3 places from five digits 2, 3, 6, 7, 8 are ${}^5P_3 = 60$

So the required numbers= $360 - 60 = 300$

220 **(b)**

Required sum= $3!\,(3+4+5+6)$

$= 6 \times 18 = 108$

221 **(c)**

Since, no two lines are parallel and no three are concurrent, therefore n straight lines intersect at ${}^nC_2 = N$ (say) points. Since, two points are required to determine a straight line, therefore the total number of lines obtained by joining N points is ${}^NC_2$. But in this each old line has been counted ${}^{n-1}C_2$times. Since, on each old line there will be $n-1$ lines.

Hence, the required number of fresh lines.

$= {}^NC_2 - n\,{}^{n-1}C_2$

$= \frac{N(N-1)}{2} - \frac{n(n-1)(n-2)}{2}$

$= \frac{{}^nC_2({}^nC_2 - 1)}{2} - \frac{n(n-1)(n-2)}{2} \quad (\because N = {}^nC_2)$

$= \frac{\frac{n(n-1)}{2}\left(\frac{n(n-1)}{2} - 1\right)}{2} - \frac{n(n-1)(n-2)}{2}$

$= \frac{n(n-1)}{8}[(n^2 - n - 2) - 4(n-2)]$

$= \frac{n(n-1)}{8}[n^2 - 5n + 6]$

$= \frac{n(n-1)(n-2)(n-3)}{8}$

222 **(c)**

The required number of ways

$= ({}^2C_1 \times {}^4C_2 + {}^2C_2 \times {}^4C_1) \times 3!$

$= (2 \times 6 + 1 \times 4)6 = 96$

223 **(a)**

The factor of $216 = 2^3 . 3^3$

The odd divisors are the multiple of 3

$\therefore$ The number of divisors=3+1=4

224 **(c)**

We have,

$E_3(100\,!) = \left[\frac{100}{3}\right] + \left[\frac{100}{3^2}\right] + \left[\frac{100}{3^3}\right]\left[\frac{100}{3^4}\right]$

$= 33 + 11 + 3 + 1 = 48$

225 **(b)**

Case I When number in two digits.

Total number of ways= $9 \times 9 = 81$

Case II When number in three digits

Total number of ways $= 9 \times 9 \times 9 = 729$

$\therefore$ Total number of ways $= 81 + 729 = 810$

226 **(b)**

We have,

${}^{56}P_{r+6} : {}^{54}P_{r+3} = 30800 : 1$

$\Rightarrow \frac{56\,!}{(50-r)\,!} = 3800\left(\frac{54\,!}{(51-r)\,!}\right)$

$\Rightarrow 56 \times 55 = \frac{3800}{51-r}$

$\Rightarrow 51 - r = 10 \Rightarrow r = 41$

227 **(c)**

We have,

Required number of numbers
= Total number of numbers formed by the digits 1,2,3,4,5
− Number of numbers having 1 at ten thousand's place
− Number of numbers having 2 at ten thousand's place and 1 at thousand's place
− Number of numbers having 2 at ten thousand's place and 3 at thousand's place
$= 5! - 4! - 3! - 3! = 120 - 24 - 6 - 6 = 84$

228 **(c)**
The number of ways of selecting 3 points out of 12 points is ${}^{12}C_3$. Three points out of 7 collinear points can be selected in ${}^7C_3$ ways
Hence, the number of triangles formed $= {}^{12}C_3 - {}^7C_3 = 185$

229 **(d)**
Required sum=(sum of the digits) $(n-1)!\left(\frac{10^n-1}{10-1}\right)$

$$= (1+2+3+4+5)(5-1)!\left(\frac{10^5-1}{10-1}\right)$$

$$= 360\left(\frac{100000-1}{9}\right) = 40 \times 99999 = 3999960$$

230 **(c)**
Total number of words formed by 4 letters form given eight different letters with repetition= 8^4
and number of words with no repetition=${}^8P_4$
∴ Required number of words= $8^4 - {}^8P_4$

231 **(d)**
Given number of flags= 5
Number of signals formed using one flag= ${}^5P_1 = 5$
Similarly, using 2 flags= ${}^5P_2$
Using 3 flags= ${}^5P_3$
Using 4 flags= ${}^5P_4$
Using 5 flags= ${}^5P_5$
∴ Total number of signals that can be formed
$= {}^5P_1 + {}^5P_2 + {}^5P_3 + {}^5P_4 + {}^5P_5$
$= 5 + 20 + 60 + 120 + 120$
$= 325$

232 **(d)**
Required number of permutations= $\frac{6!}{3!2!} = 60$

233 **(c)**
Out of 22 players 4 are excluded and 2 are to be included in every selection. This means that 9 players are to be selected from the remaining 16 players which can be done in ${}^{16}C_9$ ways

234 **(b)**
The letters in the word 'CONSEQUENCE' are 2C, 3E, 2N, 1O, 1Q, 1S, 1U
∴ Required number of permutations= $\frac{9!}{2!2!}$

235 **(c)**
The number of different sums of money Sita can form is equal to the number of ways in which she can select at least one coin out of 5 different coins
Hence, required number of ways $= 2^5 - 1 = 31$

236 **(a)**
For the first player, distribute the cards in ${}^{52}C_{17}$ ways. Now, out of 35 cards left 17 cards can be put for second player in ${}^{35}C_{17}$ ways. Similarly, for third player put them in ${}^{18}C_{17}$ways. One card for the last player can be put in ${}^1C_1$ way. Therefore, the required number of ways for the proper distribution

$$= {}^{52}C_{17} \times {}^{35}C_{17} \times {}^{18}C_{17} \times {}^1C_1$$

$$= \frac{52!}{35!\,17!} \times \frac{35!}{18!\,17!} \times \frac{18!}{17!\,1!} \times 1! = \frac{52!}{(17!)^3}$$

237 **(b)**
Suppose $x_1\,x_2\,x_3\,x_4\,x_5\,x_6\,x_7$ represents a seven digit number. Then x_1 takes the value 1,2,3, … ,9 and $x_2, x_3, \dots, x_7$ all take values 0,1,2,3, … ,9
If we keep $x_1, x_2, \dots, x_6$ fixed, then the sum $x_1 + x_2 + \cdots + x_6$ is either even or odd. Since x_7 takes 10 values 0,1,2, … 9, five of the numbers so formed will be even and 5 odd
Hence, the required number of numbers
$= 9 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 5 = 4500000$

239 **(a)**
The required number of ways is equal to the number of dearrangements of 10 objects.

∴ Required number of ways

$$= 10!\left\{1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \dots + \frac{1}{10!}\right\}$$

240 **(a)**
The number of words starting from A are =5!=120

The number of words starting from I are =5!=120

The number of words starting from KA are =4!=24

The number of words starting from KI are =4!=24

The number of words starting from KN are =4!=24

The number of words starting from KRA are =3!=6

The number of words starting from KRIA are =2!=2

The number of words starting from KRIN are =2!=2

The number of words starting from KRISA are=1!=1

The number of words starting from KRISNA are=1!=1

Hence, rank of the word KRISNA

$= 2(120) + 3(24) + 6 + 2(2) + 2(1) = 324$

241 **(a)**

Total number of words

= Number of arrangements of the letters of the word 'MATHEMATICS'

$= \frac{11\,!}{2\,!\,2\,!\,2\,!}$

242 **(c)**

We have,

$P_m = {}^mP_m = m\,!$

$\therefore 1 + P_1 + 2 \cdot P_2 + 3 \cdot P_3 + \cdots + n \cdot P_n$

$= 1 + 1 + 2 \cdot 2\,! + 3 \cdot 3\,! + \cdots + n \cdot n\,!$

$= 1 + \sum_{r=1}^{n} r \cdot (r\,!) = 1 + \sum_{r=1}^{n} \{(r+1) - 1\} r\,!$

$= 1 + \sum_{r=1}^{n} [(r+1)\,! - r\,!]$

$= 1 + \{(2\,! - 1\,!) + (3\,! - 2\,!) + (4\,! - 3\,!) + \cdots + ((n+1)\,! - n\,!)\}$

243 **(a)**

$\because \quad \frac{n(n-3)}{2} = 54$

$\Rightarrow \quad n^2 - 3n - 108 = 0$

$\Rightarrow \quad (n-12)(n+9) = 0$

$\Rightarrow \quad n = 12 \qquad [\because n \neq -9]$

244 **(c)**

In all, we have 8 squares in which 6 'X' have to be placed and It can be done in ${}^8C_6 = 28$ ways.

But this includes the possibility that either the top or horizontal row does not have any 'X'. Since, we want each row must have at least one 'X', these two possibilities are to be excluded.

Hence, required number of ways $= 28 - 2 = 26$

245 **(c)**

We have, 11 letters, viz.

$A, A; I, I; N, N; E, X; M; T; O$

For groups of 4 we may arrange these as follows:

(i) Two alike, two others alike

(ii) Two alike, two different

(iii) all four different

(i) gives rise ${}^3C_2$ selections, (ii) gives rise $3 \times {}^7C_2$ selection and (iii) gives rise ${}^8C_4$ selections

So, the number of permutations

$= 3\frac{4\,!}{2\,!\,2\,!} + 63\frac{4\,!}{2\,!} + 70.4\,! = 2454$

246 **(b)**

There are total 20+1=21 persons. The two particular persons and the host be taken as one unit so that these remain $21 - 3 + 1 = 19$ persons be arranged in round table in 18! ways. But the two persons on either sides of the host can themselves be arranged in 2! ways.

$\therefore$ Required number of ways = 2! 18! = 2.18!

247 **(d)**

${}^{n-1}C_3 + {}^{n-1}C_4 > {}^nC_3$

$\Rightarrow \quad {}^nC_4 > {}^nC_3$

$\Rightarrow \quad \frac{n!}{(n-4)!\,4!} > \frac{n!}{(n-3)!\,3!}$

$\Rightarrow \quad (n-3)(n-4)! > (n-4)!\,4$

$\Rightarrow \quad n > 7$

248 **(c)**

We know that $\frac{(m\,n)!}{(m\,!)^n}$ is the number of ways of distributing mn distinct object in n persons equally

Hence, $\frac{(mn)\,!}{(m\,!)^n}$ is a positive integer

Consequently, $(mn)\,!$ is divisible by $(m\,!)^n$

Similarly, $(mn)\,!$ is divisible by $(n\,!)^m$

Now,

$n < m \Rightarrow m + n < 2m \leq mn$ and $m - n < m < mn$

$\Rightarrow (m+n)\,!\,|(mn)\,!$ and $(m-n)\,!\,|(mn)\,!$

249 **(d)**

The number of subsets containing more than n elements is equal to

$^{2n+1}C_{n+1} + {}^{2n+1}C_{n+2} + \cdots + {}^{2n+1}C_{2n+1}$

$= \frac{1}{2}\{{}^{2n+1}C_0 + {}^{2n+1}C_1 + \cdots + {}^{2n+1}C_{2n+1}\}$

$= \frac{1}{2}(2^{2n+1}) = 2^{2n}$

251 **(a)**

We have, 9 letters $3a$'s, $2b$'s and 4 c's. These 9 letters can be arranged in $\frac{9!}{3!2!4!} = 1260$ ways

252 **(c)**

The total number of subsets of given set is $2^9 = 512$

Case I When selecting only one even number {2, 4, 6, 8}

Number of ways= ${}^4C_1 = 4$

Case II When selecting only two even numbers= ${}^4C_2 = 6$

Case III When selecting only three even numbers= ${}^4C_3 = 4$

Case IV When selecting only four even numbers = ${}^4C_4 = 1$

∴ Required number of ways

$= 512 - (4 + 6 + 4 + 1) - 1 = 496$

[here, we subtract 1 for due to the null set]

254 **(a)**

The number of ways of choosing a committee if there is no restriction is

${}^{10}C_4 \cdot {}^9C_5 = \frac{10!}{4!6!} \cdot \frac{9!}{4!5!} = 26460$

The number of ways of choosing the committee if both Mr. A and Ms. B are included in the committee is ${}^9C_3 \cdot {}^8C_4 = 5880$

Therefore, the number of ways of choosing the committee when Mr. A and Ms. B are not together $= 26480 - 5880 = 20580$

255 **(d)**

(1)It is true that product of r consecutive natural numbers is always divisible by r.

(2) Now, 115500=$2^2 \times 3^1 \times 5^3 \times 7^1 \times 11^1$

∴ Total number of proper divisor

$= (2 + 1)(1 + 1)(3 + 1)(1 + 1)(1 + 1) - 2$

$= 96 - 2 = 94$

(3) Total number of ways $= \frac{52!}{(13!)^4}$

Hence, all statements are true

256 **(d)**

Total numbers formed by using given 5 digits= $\frac{5!}{2!}$

For number greater than 40000, digit 2 cannot come at first place. Hence, number formed in which 2 is at the first place $= \frac{4!}{2!}$

Hence, total numbers formed greater than 40000

$= \frac{5!}{2!} = \frac{4!}{2!} = 60 - 12 = 48$

257 **(c)**

In the case of each book we may take 0,1,2,3, ... p copies; that is, we may deal with each book in $p + 1$ ways and therefore with all the books in $(p + 1)^n$ ways. But, this includes the case where all the books are rejected and no selection is made

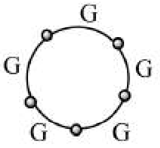
∴ Number of ways in which selection can be made

$= (p + 1)^n - 1$

258 **(b)**

First we fix the alternate position of the girls. Five girls can be seated around the circle in $(5 - 1)! = 4!$, 5 boys can be seated in five vacant place by 5!

∴ Required number of ways= $4! \times 5!$



259 **(c)**

The number of words start with $D = 6! = 720$

The number of words start with $E = 6! = 720$

The number of words start with $MD = 5! = 120$

The number of words start with $ME = 5! = 120$

Now sthe first word start with MO is MODESTY.

Hence, rank of MODESTY $= 720 + 720 + 120 + 120$

$= 1681$

260 **(c)**

Starting with the letter A and arranging the other four letters, there are $4! = 24$ words. The starting with G, and arranging A, A, I, and N in different ways, there are $\frac{4!}{2!} = 12$ words. Next the 37th word starts with I, there are 12 words starting with I. This accounts upto the 48th word. The 49th word in NAAGI. The 50th word is NAAAIG

261 **(c)**

We have the following possibilities:

Number of selections *Number of Arrangements*

$^3C_1 \times {}^2C_2$ $\quad$ $^3C_1 \times {}^2C_2 \times \frac{5!}{3!} = 60$

$^3C_2 \times {}^1C_1$ $\quad$ $^3C_2 + {}^1C_1 \times \frac{5!}{2!\,2!} = 90$

Three bottles of one type and two distinct. Two bottles of one type, two bottles of second type and one from the remaining.

Hence, required number of ways $= 60 + 90 = 150$

262 **(a)**

Six '+' signs can be arranged in a row in $\frac{6!}{6!} = 1$ way. Now, we are left with seven places in which 4 minus signs can be arranged in

$^7C_4 \times \frac{4!}{4!} = 35$

263 **(b)**

$\because$ The candidate is unsuccessful, if he fails in 9 or 8 or 7 or 6 or 5 papers.

$\therefore$ Numbers of ways to be unsuccessful

$= {}^9C_9 + {}^9C_8 + {}^9C_7 + {}^9C_6 + {}^9C_5$

$= {}^9C_0 + {}^9C_1 + {}^9C_2 + {}^9C_3 + {}^9C_4$

$= \frac{1}{2}({}^9C_0 + {}^9C_1 + \ldots\ldots + {}^9C_9)$

$= \frac{1}{2}(2^9) = 2^8 = 256$

264 **(d)**

Using the digits 0, 1, 2, ... ,9 the number of five digits telephone numbers which can be formed is 10^5(since repetition is allowed).

The number of five digits telephone numbers, which have none of digits repeated $= {}^{10}P_5 = 30240$

$\therefore$ The required number of telephone number

$= 10^5 - 30240 = 69760$

265 **(d)**

The number of words beginning with 'a' is same as the number of ways of arranging the remaining 4 letters taken all at a time. Therefore 'a' will occur in the first place 4 ! times. Similarly, b or c will occur in the first place the same number of times. Then, d occurs in the first place. Now, the number of words beginning with 'da, db or dc' is 3 !. Then, the words beginning with 'de' must follow. The first one is '$de\ abc$', the next one is '$de\ acb$' and the next to the next comes '$de\ bac$'. So, the rank of '$de\ bac$' $= 3 \cdot 4! + 3 \cdot 3! + 3 = 93$

266 **(b)**

Required number $= 2\ {}^{20}C_2$

267 **(c)**

The total number of combinations which can be formed of five different green dyes, taking one or more of them is $2^5 - 1 = 31$. Similarly, by taking one or more of four different red dyes $2^4 - 1 = 15$ combinations can be formed. The number of combinations which can be formed of three different red dyes, taking none, one or more of them is $2^3 = 8$

Hence, the required number of combinations of dyes

$= 31 \times 15 \times 8 = 3720$

268 **(b)**

We observe that

4 lines intersect each other in ${}^4C_2 = 6$ points

4 circles intersect each other in ${}^4C_2 \times 2 = 12$ points

A line cuts a circle in 2 points

$\therefore$ 4 lines will cut four circles into $2 \times 4 \times 4 = 32$ points

Hence, required number of points $= 6 + 12 + 32 = 50$

269 **(a)**

From the given relation it is evident that ${}^nC_r$ is the greatest among the values ${}^nC_0, {}^nC_1, \ldots, {}^nC_n$

We know that ${}^nC_r$ is greatest for $r = \frac{n}{2}$. Hence, $r = \frac{n}{2}$

270 **(d)**

A committee may consists of all men and no women or all women and no men or 3 men and 1 women whose is not among wives of 3 chosen men or, 2 men and 2 women who are not are not among the wives of 2 chosen men or 1 men and 3 women none of whom is wife of chosen men

$\therefore$ Required number of committees

$= {}^4C_4 + {}^4C_4 + {}^4C_3 \times {}^1C_1 + {}^4C_2 \times {}^2C_2 + {}^4C_1 \times {}^3C_3 = 16$

271 **(d)**

The women choose the chairs amongst the chairs marked 1 to 4 in ${}^4P_2$ways and the men can select the chairs from remaining in ${}^6P_3$ ways

Total number of ways $= {}^4P_2 \times {}^6P_3$

272 **(a)**

Let n be the number of diagonals of a polygon.

Then, ${}^nC_2 - n = 44$

$\Rightarrow \frac{n(n-1)}{2} - n = 44$

$\Rightarrow n^2 - 3n - 88 = 0$

$\Rightarrow n = -8$ or 11

$\therefore n = 11$

273 **(b)**

In the word '*exercises*' there are 9 letters of which 3 are e's and 2 are s's

So, required number of permutations $= \frac{9!}{3!2!} = 30240$

274 **(b)**

Total number of functions

=Number of dearrangement of 5 objects

$= 5!\left(\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!}\right) = 44$

275 **(a)**

The total number of ways in which words with five letters are formed from given 10 letters $= 10^5 = 100000$

Total number of ways in which words with five letters are formed (no repetition) $= 10 \times 9 \times 8 \times 7 \times 6 = 30240$

$\therefore$ Required number of ways $= 100000 - 30240 = 69760$

277 **(d)**

We have,

Required number of numbers = Number of three digit numbers divisible by 5 + number of 4 digit numbers divisible by 5

$= {}^3C_2 \times 2! \times 1 + ({}^3C_3 \times 3!) \times 1 = 6 + 6 = 12$

279 **(c)**

In 8 squares $6x$ can be placed in 28 ways but there are two methods in which there is no x in first or last row.

$\therefore$ required number of ways=28-2=26

280 **(d)**

Total number of points on a three lines are $m + n + k$

$\therefore$ maximum number of triangles

$= {}^{m+n+k}C_3 - {}^mC_3 - {}^nC_3 - {}^kC_3$

(subtract those triangles in which point on the same line)

281 **(d)**

In the word RAHUL the letters are (A, H, L, R, U)

Number of words starting with A= $4! = 24$

Number of words starting with H=$4! = 24$

Number of words starting with L= $4! = 24$

In the starting with R first one is RAHLU and next one is RAHUL.

$\therefore$ Rank of the word RAHUL= $3(24) + 2 = 74$

282 **(a)**

The required natural numbers consist of 4 digits, 3 digits, 2 digits and one digit so that their number is equal to

$9 \cdot 9 \cdot 8 \cdot 7 + 9 \cdot 9 \cdot 8 + 9 \cdot 9 + 9 = 5274$

283 **(a)**

We have 26 letters (a to z) and 10 digits (0 to 9). The first three places can be filled with letters in ${}^{26}P_3$ ways and the remaining 2 places can be filled with digits ${}^{10}P_2$ ways. Hence, the number of ways in which the code word can be made

$= ({}^{26}C_3 \times 3!) \times ({}^{10}C_2 \times 2!) = 1404000$

284 **(c)**

The first digit a can take any one of 1 to 8

The third digit c can take any one of 0 to 9

When a =1, b can take any one of 2 to 9=8 values

When $a = 2$, b can take any one of 3 to 9=7 values

When $a = 3$, b can take any one of 4 to 9=6 values

… … … …

… … … …

When $a = 8$, b can take any one $(b = 9) = 1$ values Thus, the number of total numbers

$= (8+7+6+\ldots+2+1)\times 10 = \frac{8\times 9}{2}\times 10$

$= 360$

285 **(c)**

Since, out of eleven members two numbers sit together, then the number of arrangements= $9! \times 2$

($\because$ Two numbers can be sit in two ways)

286 **(c)**

There are 4 odd places and there are 4 odd numbers viz. 1, 1, 3, 3. These, four numbers can be arranged in four places in

$\frac{4!}{2!\,2!} = 6$ ways

In a seven digit are 3 even places namely 2nd, 4th and 6th in which 3 even numbers 2, 2, 4 can be arranged in $\frac{3!}{2!} = 3$ ways

Hence, the total number of numbers $= 6\times 3 = 18$

287 **(d)**

The number of words starting from E are =5!=120

The number of words starting from H are =5!=120

The number of words starting from ME are=4!=24

The number of words starting from MH are=4!=24

The number of words starting from MOE are =3!=6

The number of words starting from MOH are =3!=6

The number of words starting from MOR are =3!=6

The number of words starting from MOTE are =2!=2

The number of words starting from MOTHER are =1!=1

Hence, rank of the word MOTHER

$= 2(120) + 2(24) + 3(6) + 2 + 1$

$= 309$

288 **(c)**

(1) Total number of ways of arranging m things = $m!$ To find the number of ways in which p particular things are together, we consider p particular thing as a group.

$\therefore$ Number of ways in which p particular things are together $= (m-p+1)!\,p!$

So, number of ways in which p particular things are not together

$= m! - (m-p+1)!\,p!$

(2) Each player shall receive 13 cards.

Total number of ways $= \frac{52!}{(13!)^4}$

Hence, both statements are correct

289 **(d)**

Now, 770=2.5.7.11

We can assigned 2 to $x_1$or $x_2$or x_3 or x_4. That is 2 can be assigned in 4 ways.

Similarly each of 5, 7 or 11 can be assigned in 4 ways.

$\therefore$ Required number of ways $= 4^4 = 256$

290 **(c)**

There are five seats in a bus are vacant. A man sit on any one of 5 seats in 5 ways. After the man is seated his wife can be seated in any of 4 remaining seats in 4 ways.

Hence, total number of ways of seating them= $5\times 4 = 20$

291 **(c)**

Required number = ${}^9C_5 - {}^7C_3 = 91$

292 **(b)**

Since, there are n distinct points on a circle.

For making a pentagon it requires a five points

According to given condition

${}^nC_5 = {}^nC_3 \Rightarrow n = 8$

293 **(b)**

The total number of ways= $6^4 = 1296$

$\therefore$ required number of ways

=1296-(none of the number shows 2)

$= 1296 - 5^4 = 671$

294 **(c)**

Required number of ways

$= {}^{11}C_5 - {}^{11}C_4$

$= \frac{11!}{5!\,6!} = \frac{11!}{4!\,7!} = 132$

295 **(d)**

There are $(m+1)$ choices for each of n different books. So, the total number of choice is $(m+1)^n$ including one choice in which we do not select any book.

Hence, the required number of ways is $(m+1)^n - 1$

296 **(b)**

There are 6 letters in the word 'MOBILE'. Consequently, there are 3 odd places and 3 even places. Three consonants M, B and L can occupy three odd places in 3 ! ways. Remaining three places can be filled by 3 vowels in 3 ! ways.

Hence, required number of words $= 3! \times 3! = 36$

297 **(b)**

As the seats are numbered so the arrangement is not circular

Hence, required number of arrangements $= {}^nC_m \times m!$

298 **(d)**

Two circles can intersect at most in two points. Hence, the maximum number of points of intersection is ${}^8C_2 \times 2 = 56$

299 **(b)**

There are two cases arise

Case I They do not invite the particular friend

$= {}^8C_6 = 28$

Case II They invite one particular friend

$= {}^8C_5 \times {}^2C_1 = 112$

$\therefore$ Required number of ways =28+112=140

300 **(d)**

The consonants can be arranged in 4! ways, and the vowels in $\frac{3!}{2!}$ ways

So, the required number of arrangements $= \frac{4!\,3!}{2}$

301 **(c)**

$\because$ Each true-false questions can be answered in 2 ways

$\therefore$ Number of ways in which 10 questions can be answered

$= 2^{10} = 1024$

302 **(b)**

The required number of ways $= {}^{8-1}C_{3-1} = \frac{7!}{2!5!} = 21$

303 **(c)**

The total number of two factor products $= {}^{100}C_2$. Out of the numbers 1,2,3, ... ,100; the multiples of 3 are 3,6,9, ... ,99 i.e., there are 33 multiples of 3, and therefore there are 67 non-multiples of 3

So, the number of two factor products which are not multiples of 3 $= {}^{67}C_2$

So, the required number $= {}^{100}C_2 - {}^{67}C_2 = 2739$

304 **(c)**

Since each question can be dealt with in 3 ways, by selecting it or by selecting its alternative or by rejecting it. Thus, the total number of ways of dealing with 10 given questions is 3^{10} including a way in which we reject all the questions

Hence, the number of all possible selections $= 3^{10} - 1$

306 **(c)**

The number of ways in which four different balls can be placed in four different boxes

$= {}^4C_1 + {}^3C_1 + {}^2C_1 + {}^1C_1$

$= 4 + 3 + 2 + 1 = 10$

$\therefore$ Required number of ways $= 10 - 1 = 9$

[Since only one way in which the same ball have a same box]

307 **(b)**

First stall can be filled in 3 ways, second stall can be filled in 3 ways and so on.

$\therefore$ Number of ways of loading steamer

$= 3 \times 3 \times \ldots \times 3$(12 times) $= 3^{12}$

309 **(a)**

Five boys can be seated in a row in 5 ! ways. There are 4 places between five boys in which 3 girls can be seated in ${}^4C_3 \times 3!$ ways

Hence, required number of ways $= 5! \times {}^4C_3 \times 3! = 2880$

310 **(b)**

$\because$ Each letter can be posted in 3 ways

$\therefore$ Total number of ways $= 3^6$

311 **(a)**

$\because$ 26 cards can be chosen out of 52 cards in $^{52}C_{26}$ ways. There are two ways in which each card can be dealt because a card can be either from the first pack or from the second.

$\therefore$Total number of ways = $^{52}C_{26}.2^{26}$

313 **(a)**

Given ,

$^{8}C_r - {^7C_3} = {^7C_2}$

$\Rightarrow {^8C_r} = {^7C_3} + {^7C_2}$

$\Rightarrow {^8C_r} = {^8C_3}$

$\Rightarrow r = 3$

315 **(c)**

If the last two digits are 0, 0 then in 1st digit any of the numbers expect 0 *ie*, 9 numbers

If the last two digits are 1, 1, then in 1st digit any of the numbers expect 0 and 1 *ie*, 8 numbers

$\therefore$ The total number of numbers= $9 + 8 \times 9 = 81$

316 **(a)**

For A, B, C to speak in order of alphabets, 3 places out of 10 may be chosen first in $^{10}C_3$ ways. The remaining 7 persons can speak in 7! ways. Hence, the number of ways in which all the 10 persons can speak = $^{10}C_3.7! = \frac{10!}{3!} = \frac{10!}{6}$

318 **(b)**

Since, a student is allowed to select at most n books out of $(2n + 1)$*books*

If T is the total number of ways selecting one book, then

$T = {^{2n+1}C_1} + {^{2n+1}C_2} + \ldots + {^{2n+1}C_n} = 63$ ...(i)

Using the binomial theorem

$^{2n+1}C_0 + {^{2n+1}C_1} + \ldots + {^{2n+1}C_n} + {^{2n+1}C_{n+1}} + \ldots$

$= (1 + 1)^{2n+1} = 2^{2n+1}$

$\Rightarrow {^{2n+1}C_0} + 2({^{2n+1}C_1} + {^{2n+1}C_2} + \cdots + {^{2n+1}C_n})$
$+ {^{2n+1}C_{2n+1}} = 2^{2n+1}$

$\Rightarrow 1 + 2(63) + 1 = 2^{2n+1}$

$\Rightarrow \quad 1 + 63 = \frac{2^{2n+1}}{2} = 2^n$

$\Rightarrow \quad 2^6 = 2^{2n} \Rightarrow \quad n = 3$

319 **(a)**

For each historical monument, there are two possibilities either he visit or does not visit.

Total number of ways = $2^5.2^6(2^7 - 1)$

320 **(d)**

The number of divisors of ab^2c^2de

$= (1 + 1)(2 + 1)(2 + 1)(1 + 1)(1 + 1) - 1$

$= 2.3.3.2.2. - 1 = 71$

321 **(c)**

When we arrange one things at a time, the number of possible permutations is n. When we arrange them two at a time the number of possible permutations are $n \times n = n^2$

and so on. Thus, the total number of permutations are

$$n + n^2 + \ldots . + n^r = \frac{n(n^r - 1)}{n - 1} \quad (\because n > 1)$$

323 **(c)**

Given, $a_n = \sum_{r=0}^{n} \frac{1}{^nC_r}$

Let $b_n = \sum_{r=0}^{n} \frac{r}{^nC_r}$

Then, $b_n = \frac{0}{^nC_0} + \frac{1}{^nC_1} + \frac{2}{^nC_2} + \ldots \ldots + \frac{n}{^nC_n}$ ...(i)

$\Rightarrow b_n = \frac{n}{^nC_0} + \frac{n-1}{^nC_1} + \frac{n-2}{^nC_2} + \ldots \ldots + \frac{0}{^nC_n}$ ...(ii)

$[\because {^nC_0} = {^nC_n}, {^nC_1} = {^nC_{n-1}} \ldots . \text{as } {^nC_r} = {^nC_{n-r}}]$

On adding Eqs. (i) and (ii), we get

$$2b_n = \frac{n}{^nC_0} + \frac{n}{^nC_1} + \ldots \ldots + \frac{n}{^nC_n}$$

$$= n\left[\frac{1}{^nC_0} + \frac{1}{^nC_1} + \frac{1}{^nC_2} + \ldots \ldots + \frac{1}{^nC_n}\right]$$

$\Rightarrow 2b_n = na_n$

$$\therefore b_n = \frac{1}{2}na_n$$

324 **(b)**

Number of five digit numbers that can be formed by using the digits 3, 4 and 7 and 5 is used twice= $\frac{5!}{2!} = 60$

325 **(c)**

The number of words begin with A=4!=24

The number of words begin with G = $\frac{4!}{2!} = 12$

The number of words begin with I = $\frac{4!}{2!} = 12$

So, 49th and 50th words begin with N and in dictionary order 49th is NAAGI and 50th will be NAAIG

326 **(c)**
There are two possible cases
Case I Five 1's, one 2's, one 3's
Number of numbers= $\frac{7!}{5!} = 42$
Case II Four 1's, three 2's
Number of numbers= $\frac{7!}{4!3!} = 35$
Total number of numbers $42 + 35 = 77$

327 **(a)**
A polygon of n sides has number of diagonals
$= \frac{n(n-3)}{2} = 275$ [given]
$\Rightarrow n^2 - 3n - 550 = 0$
$\Rightarrow (n-25)(n+22) = 0$
$\Rightarrow n = 25 \quad [\because n \neq -22]$

328 **(d)**
In the word MATHEMATICS the letters are 2A, C, E, H, I, 2M, S, 2T
$\therefore$ Total number of different words= $\frac{11!}{2!2!2!} = \frac{11!}{(2!)^3}$

329 **(a)**
First we fix the alternate position of English books. Then there are 22 vacant places for Hindi books.

Hence, total number of ways = ${}^{22}C_{19} = \frac{22!}{3!9!} =$ 1540

330 **(d)**
The boys are in majority, if the groups are 4B, 3G, 5B, 2G, 6B 1G
Total number of combinations
$= {}^6C_4 \times {}^4C_3 + {}^6C_5 \times {}^4C_2 + {}^6C_6 \times {}^4C_1$
$= 15 \times 4 + 6 \times 6 + 1 \times 4 = 100$

331 **(a)**
Given, ${}^nC_r = {}^nC_{r-1}$ and ${}^nP_r = {}^nP_{r+1}$

$$\Rightarrow \frac{n!}{(n-r)!\,r!} = \frac{n!}{(n-r+1)!\,(r-1)!}$$

and $\frac{n!}{(n-r)!} = \frac{n!}{(n-r-1)!}$

$\Rightarrow n - r + 1 = r$ and $n - r = 1$

$\Rightarrow n - 2r + 1 = 0$ and $n - r - 1 = 0$

On solving, we get

$n = 3, r = 2$

332 **(a)**
For length, number of choices is

$(2m-1) + 2m - 3) + \ldots + 3 + 1 = m^2$

Similarly, for breadth number of choices is

$(2n-1) + (2n-3) + \ldots + 3 + 1 = n^2$

Hence, required number of choices is m^2n^2

333 **(b)**
If L is middle, then first two places can be filled by ${}^4P_2$ ways and the last two digits can be filled in 2! Ways.

$\therefore$ Required number of ways = ${}^4P_2 \times 2!$

$= 12 \times 2 = 24$

334 **(c)**
Here, we are concerned with mere grouping and the number of persons in each group is same
$\therefore$ Required number of ways = $\frac{12!}{(4!)^3\,3!}$

336 **(c)**
The hall can be illuminated by switched on at least one of the 10 bulbs. Therefore, the required number of ways is $2^{10} - 1 = 1023$

337 **(a)**
Given, $s_n = \sum_{r=0}^{n} \frac{1}{{}^nC_r} = \sum_{r=0}^{n} \frac{1}{{}^nC_r} \quad [\because {}^nC_r = {}^nC_r]$

$$\Rightarrow ns_n = \sum_{r=0}^{n} \frac{n}{{}^nC_{n-r}} = \sum_{r=0}^{n} \left[\frac{n-r}{{}^nC_{n-r}} + \frac{r}{{}^nC_{n-r}}\right]$$

$$\Rightarrow ns_n = \sum_{r=0}^{n} \frac{n-r}{{}^nC_{n-r}} + \sum_{r=0}^{n} \frac{r}{{}^nC_r}$$

$$\Rightarrow ns_n = \left(\frac{n}{{}^nC_n} + \frac{n-1}{{}^nC_{n-1}} + \ldots + \frac{1}{{}^nC_0}\right) \sum_{r=0}^{n} \frac{r}{{}^nC_r}$$

$\Rightarrow ns_n = t_n + t_n = 2t_n$

$\Rightarrow \frac{t_n}{s_n} = \frac{n}{2}$

338 **(c)**
Here, $\frac{{}^nC_{r-1}}{{}^nC_r} = \frac{36}{84}$ and $\frac{{}^nC_r}{{}^nC_{r+1}} = \frac{84}{126}$
$\Rightarrow 3n - 10r = -3$
and $4n - 10r = 6$
On solving, we get $n = 9$ and $r = 3$

339 **(b)**
Given word is HAVANA (3A, 1H, 1N, 1V)
Total number of ways arranging the given word

$= \frac{6!}{3!} = 120$

Total number of words in which N, V together

$= \frac{5!}{3!} \times 2! = 40$

∴ Required number of ways $= 120 - 40 = 80$

340 **(c)**

Rank of word in a dictionary

$= 2 \times 5! + 2 \times 4! + 3 \times 3! + 2! + 1$

$= 240 + 48 + 18 + 2 + 1$

$= 309$

342 **(d)**

We have,

${}^nC_r + 2\,{}^nC_{r-1} + {}^nC_{r-2}$

$= ({}^nC_r + {}^nC_{r-1}) + ({}^nC_{r-1} + {}^nC_{r-2})$

$= {}^{n+1}C_r + {}^{n+1}C_{r-1} = {}^{n+2}C_r$

343 **(a)**

We have,

$r \cdot {}^{n-1}P_{r-1} + {}^{n-1}P_r = {}^nP_r$

$\Rightarrow 6 \cdot {}^{11}P_5 + {}^{11}P_6 = {}^{12}P_6$

$\Rightarrow {}^{12}P_r = {}^{12}P_6 \Rightarrow r = 6$ $[\because 6 \cdot {}^{11}P_5 + {}^{11}P_6 = {}^{12}P_r \text{(given)}]$

344 **(a)**

A selection of 3 balls so as to include at least one black ball, can be made in the following 3 mutually exclusive ways

(i) The number of ways in which 1 black balls and 2 others are selected

$= {}^3C_1 \times {}^6C_2 = 3 \times 15 = 45$

(ii) The number of ways in which 2 black balls and 1 other are selected

$= {}^3C_2 \times {}^6C_1 = 3 \times 6 = 18$

(iii) The number of ways in which 3 black balls and no other are selected $= {}^3C_3 = 1$

∴ Total numbers of ways $= 45 + 18 + 1 = 64$

345 **(b)**

A triangle is obtained by joining three non-collinear point

∴ The total number of triangles $= {}^{18}C_3 - {}^5C_3 = 806$

346 **(b)**

Suppose he invites r friends at a time. Then the total number of parties is $20C_r$. We have to find the maximum value of $20C_r$, which is for $r = 10$ (if n is even, then ${}^nC_r$ is maximum for $r = n/2$).

Hence, he should invite 10 friends at a time in order to form the maximum number of parties

347 **(c)**

Required number of diagonals $= {}^mC_2 - m$

$= \frac{m(m-1)}{2!} - m$

$= \frac{m}{2!}(m-3)$

348 **(b)**

${}^nP_r = {}^nC_r\, r!$

$\Rightarrow \frac{{}^nP_r}{r!} = {}^nC_r$

$\Rightarrow \sum_{r=1}^{n} \frac{{}^nP_r}{r!} = \sum_{r=1}^{n} {}^nC_r = {}^nC_1 + {}^nC_2 + \cdots + {}^nC_n$

$= 2^n - 1$

349 **(c)**

Four letters can be selected in the following ways

(i) all different i. e. C, O, R, G

(ii) 2 like and 2 different i.e. two O, 1 R and 1 G

(iii) 3 like and 1 different i.e. three O and 1 from R, G and C

The number of ways in (i) is ${}^4C_4 = 1$

The number of ways in (ii) is ${}^3C_2 \cdot {}^2C_2 = 3$

The number of ways in (iii) is ${}^3C_3 \times {}^3C_1 = 3$

So, the required number of ways $= 1 + 3 + 3 = 7$

350 **(d)**

∵ Number are either all even or one even and other two odd

∴ Required number of ways $= {}^{15}C_3 + {}^{15}C_1 \times {}^{15}C_2$

$= \frac{15!}{3! \times 12!} + \frac{15!}{14!} \times \frac{15!}{2! \times 13!}$

$= \frac{15 \times 14 \times 13}{6} + \frac{15 \times 15 \times 14}{2}$

$= 455 + 1575 = 2030$

351 **(b)**

Three letters can be posted in 4 letter boxes in $4^3 = 64$ ways but it consists the 4 ways that all letters may be posted in same box.

Hence, required number of ways $= 64 - 4 = 60$

352 **(c)**

The number of ways of selecting 3 points out of 12 points is $^{12}C_3$. The number of ways of selecting 3 points out of 7 points on the same straight line is $^7C_3$.
Hence, the number of triangles formed $= {}^{12}C_3 - {}^7C_3 = 185$

353 **(a)**

Required number of selections

$^3C_1 \times ^4C_1 \times ^2C_1(^6C_3 + ^6C_2 + ^6C_1 + ^6C_0)$

$= 3 \times 4 \times 2(20 + 15 + 6 + 1) = 42(4!)$

354 **(b)**

The digit x_1 can be selected in 9 ways as 0 cannot be selected. The digit x_2 can be selected in 9 ways as 0 can selected but digit in position x_1 cannot be selected. Similarly, all the remaining digits can also be selected in 9 ways.

Hence, total number of ways $= 9^n$

355 **(d)**

We have,

$^{n-1}C_6 + ^{n-1}C_7 > ^nC_6$

$\Rightarrow ^nC_7 > ^nC_6$

$\Rightarrow \frac{n!}{(n-7)!\,71} > \frac{n!}{(n-6)!\,6!}$

$\Rightarrow \frac{1}{7} > \frac{1}{n-6} \Rightarrow n > 13$

356 **(b)**

We have the following ways of selections:

$p-identical\ things$	$q-identical\ things$	$Number\ of\ ways$
p	$r-p$	1
$p-1$	$r-(p-1)$	1
:	:	:
:	:	:
$r-q$	q	1

$\therefore$ Total number of ways $= p - (r-q) + 1$ or, $q - (r-p) + 1$

$= p + q - r + 1$

357 **(d)**

Since, S_1 speak after S_2, Therefore two places can be chosen out of 10 place in $^{10}C_2$ ways and rest of the 8 speakers can speak in 8! ways.

$\therefore$ Required number of ways $= {}^{10}C_2.8!$

$= \frac{10!}{2!\,8!}.8! = \frac{10!}{2!}$

358 **(a)**

12 persons can be seated around a round table in 11 ! ways. The total number of ways in which 2 particular persons sit side by side is 10 ! × 2 !.

Hence, the required number of arrangements

$= 11! - 10! \times 2! = 9 \times 10!$

359 **(b)**

Total number of ways $= {}^5C_4 \times {}^8C_6 \times {}^5C_5 \times {}^8C_5$

$= \frac{5!}{4! \times 1!} \times \frac{8!}{2! \times 6!} + \frac{8!}{5! \times 3!}$

$= \frac{5 \times 8 \times 7}{2} + \frac{8 \times 7 \times 6}{6}$

$= 140 + 56 = 196$

360 **(b)**

Total number of points are $m + n + k$, the triangles formed by these points $= {}^{m+n+k}C_3$

Joining of three points on the same line gives no triangle, the number of such triangles is $^mC_3 + {}^nC_3 + {}^kC_3$

$\therefore$ Required number of triangles

$= {}^{m+n+k}C_3 - {}^mC_3 - {}^nC_3 - {}^kC_3$

361 **(b)**

Total number of selections $= {}^4C_3 \times {}^5C_3 \times {}^6C_3$

$= 4 \times 10 \times 20 = 800$

362 **(b)**

Total number of books $= a + 2b + 3c + d$

$\therefore$ The Total number of arrangements

$= \frac{(a + 2b + 3c + d)!}{a!\,(b!)^2(c!)^3}$

363 **(a)**

Given that, $^{n+2}C_8 : {}^{n-2}P_4 = \frac{57}{16}$

$\Rightarrow \frac{(n+2)(n+1)n(n-1)}{1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8} = \frac{57}{16}$

$\Rightarrow (n+2)(n+1)n(n-1)$

$= \frac{57}{16} \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8$

$= 57 \times 3 \times 4 \times 5 \times 6 \times 7$
$= 21 \times 20 \times 19 \times 18$
$\Rightarrow n = 19$

364 **(b)**

$\frac{1}{{}^4C_n} = \frac{1}{{}^5C_n} + \frac{1}{{}^6C_n}$

$\Rightarrow \frac{n!\,(4-n)!}{4!} = \frac{n!\,(5-n)!}{5!} + \frac{n!\,(6-n)!}{6!}$

$\Rightarrow \frac{(4-n)!}{4!} = \frac{(4-n)!\,(5-n)}{5 \times 4!} + \frac{(6-n)(5-n)(4-n)!}{6 \times 5 \times 4!}$

$\Rightarrow 1 = \frac{5-n}{5} + \frac{(6-n)(5-n)}{6 \times 5}$

$\Rightarrow n^2 - 17n + 30 = 0$

$\Rightarrow (n-15)(n-2) = 0$

$\Rightarrow n = 2$

[$\because$ ${}^4C_n$ is not meaningful for $n = 15$]

365 **(b)**

$\because \ {}^nP_r = 1680$

$\Rightarrow \frac{n!}{(n-r)!} = 1680$ ...(i)

And ${}^nC_r = 70$

$\Rightarrow \frac{n!}{(n-r)!r!} = 70$ ...(ii)

From Eqs. (i) and (ii), we get

$r! = \frac{1680}{70} = 24 = 4! \Rightarrow r = 4$

On putting the value of r in Eq. (i), we get

$\Rightarrow \frac{n!}{(n-4)!} = 1680$

$\Rightarrow n(n-1)(n-2)(n-3) = 1680$

$\Rightarrow n(n-1)(n-2)(n-3) = 8(8-1)(8-2)(8-3)$

$\Rightarrow n = 8$

$\therefore 69n + r! = 69 \times 8 + 24 = 576$

366 **(b)**

Each digit can be placed in 2 ways.

$\therefore$ Required number of ways $= 2^{10}$

367 **(d)**

$1 + 1.P_1 + 2.P_2 + 3.P_3 + \cdots + n.P_n$
$= 1 + 1.(1!) + 2.(2!) + \cdots + n(n!)$
$= 1 + (2-1)1! + (3-1)2! + \cdots + \big((n+1)-1\big)n!$
$= 1 + 2! - 1! + 3! - 2! + \ldots + (n+1)! - n!$
$= (n+1)!$

368 **(a)**

We have,

$\frac{{}^{2n+1}P_{n-1}}{{}^{2n-1}P_n} = \frac{3}{5}$

$\Rightarrow 5 \cdot {}^{2n+1}P_{n-1} = 3 \cdot {}^{2n-1}P_n$

$\Rightarrow 5 \cdot \frac{(2n+1)!}{(n+2)!} = \frac{3(2n-1)!}{(n-1)!}$

$\Rightarrow \frac{5(2n+1)(2n)(2n-1)!}{(n+2)(n+1)n\,(n-1)!} = \frac{3 \cdot (2n-1)!}{(n-1)!}$

$\Rightarrow 10(2n+1) = 3(n+2)(n+1)$

$\Rightarrow 3n^2 - 11n - 4 = 0 \Rightarrow n = 4$

369 **(c)**

From the number 112233, the number of 6 digits that can be formed the digits$= \frac{6!}{2!2!2!} = \frac{720}{8} = 90$

370 **(c)**

Since, r, s, t are prime numbers

$\therefore$ Selection of p and q are as under

p	q	Number of ways
r^0	r^2	1 way
r^1	r^2	1 way
r^2	r^0, r^1, r^2	3 way

$\therefore$ Total number of ways to select $r = 5$

s^0	s^4	1 way
s^1	s^4	1 way
s^2	s^4	1 way
s^3	s^4	1 way
s^4	s^0, s^1, s^2, s^3, s^4	5 ways

$\therefore$ Total number of ways to select $s = 9$

Similarly, the number of ways to select $t = 5$

$\therefore$ Total number of ways$5 \times 9 \times 5 = 225$

371 **(a)**

In a given word 'MAXIMUM', vowels (A, I, U) fix the alternate position in 3! Ways.

And last of the consonants (M, M, M, X) in four places can be placed in $\frac{4!}{3!}$ ways.

$\therefore$ Required number of ways $= 3! \times \frac{4!}{3!} = 4!$ ways